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The Relation between the Specular and the Diffuse Photographic Densities — LUDWIK SILBERSTEIN AND CLIFTON TUTTLE 365
Unidirectional Quanta in Wave Mechanics — G. BREIT 374

INSTRUMENT SECTION

Editorial — 381
Some Experimental Methods in Magnetostriction — S. R. WILLIAMS 383
A Calibrated Thermal Resistance Plate for Measuring Heat Flow — CARL G. F. ZOBEL 409
Velocity Selector for Atomic Rays — J. TYKOCINSKI-TYKOCINER 423
An Application of the Vacuum Tube Oscillator — C. B. CROFUTT 431
A General Expression for the Temperature Correction Based on Regnault's Method — C. H. DWIGHT 433
A New Type of Photographic Exposure Meter and Photometer — F. H. NORTON 435
The Extrusion of Bismuth Wire — DONALD C. STOCKBARGER 445
Note on the Production of Fluorite in the Electric Furnace — DONALD C. STOCKBARGER 448
Reviews — 434
Notes — 382, 408, 432
Index to Advertisers — Cover III

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THE RELATION BETWEEN THE SPECULAR AND THE DIFFUSE PHOTOGRAPHIC DENSITIES*

By Ludwik Silberstein and Clifton Tuttle

The density of a photographic plate or film is defined as the common logarithm of its opacity, which is the reciprocal of the transparency. Since, as was already pointed out by Abney, along with the screening off or absorption the scattering of light by the developed silver grains plays a considerable role, the values of density obtained will be different according as the total transmitted light, e.g. with the aid of an integrating sphere, or only that emerging normally to the plate is measured. In the former case we speak of *diffuse*, and in the latter of *specular* density. The incident light is here assumed to be collimated (or nearly so) in both cases, in accordance with the actual conditions of our measurements.[1] The specular density will, clearly, be greater than the diffuse one. According to Callier, for the materials measured by him, the ratio of the two densities would be constant throughout a wide range. Such, however, is only approximately the case. Their relation is more accurately expressed by an equation of logarithmic form proposed by Renwick and Bloch in a purely empirical way. This subject has been treated experimentally by one of us in a recent paper (J.O.S.A., & R.S.I.), *12*, p. 559; 1926) where also some references will be found.

* Communication No. 297 from the Research Laboratory of the Eastman Kodak Company. Paper read at the joint meeting of the Optical Society of America with the American Physical Society at New York, February 25, 1927.

[1] An alternative treatment (applied, e. g., by Callier) consists in recording in both cases only the normally emergent light flux but using diffused incident light (opal glass) in one and collimated incident light in the other case. The former is also the method usually employed in densitometry.

The purpose of the present paper is to derive a formula correlating the two densities from general theoretical considerations and to test it experimentally.

Let us first write down the definitions of the specular and the diffuse density, to be henceforth denoted by D and D' respectively.[2] If I_0 be the intensity of the incident and I that of the emergent direct (collimated) light, or briefly the incident and the directly transmitted flux, and I_s the total emergent flux of scattered light, then the diffuse density is, with log written for $\log_{10}$,

$$D' = \log [I_0/(I+I_s)] \tag{1}$$

and the specular density,

$$D = \log [I_0/(I+\epsilon I_s)], \tag{2}$$

where ϵ is the fraction of the total scattered flux contained in the normally or quasi-normally emerging beam. The numerical value of this fraction which would depend in a complicated way on the grain size and the wave length will be left free, to be determined in each case as an empirical constant.[3] Our object is to correlate the densities thus defined.

Part of the incident light is scattered, part is absorbed by the grains, and the remainder is directly transmitted. Let, for the moment, I be the directly transmitted intensity at any depth x of the layer. Then the energy scattered, per unit time, by a single grain placed at the depth x and struck by this light can be put equal to AI. For particles small compared with the wave length A would be proportional to the square of their volume and inversely proportional to the fourth power of the wave length λ. In our case, the size of the grain being of the order of and even a few times larger than λ, the scattering coefficient A will be a much more complicated function of λ and grain size, whose form is far from being known.[4] It will here be considered as a coefficient characterizing the average grain of the given material. Its dimensions are those of an area.

[2] Replacing the usual but less convenient symbols $d_{\parallel}$ and $d^{\dagger\dagger}$.

[3] The fraction ϵ might be a function of I_s itself. But it will appear that the observations are well represented by attributing to ϵ, for each given material, a constant value.

[4] One might think of applying Mie's scattering formulas (Annalen der Physik, *25*, p. 377, 1908). These, of course, are derived for spherical particles only. But even if one wished to disregard the departure from sphericity, there would still remain the difficulty that the two optical constants appearing in Mie's formulas are not known for the spongelike state of silver as it occurs in the developed grains and can scarcely be identified with those found for metallic silver.

If N be the number of grains per unit volume at a depth x of the layer and if the grains are supposed to be equal (i.e. to represent the average of the actual unavoidably different silver grains), the energy scattered in a stratum of thickness dx and of unit cross section will be

$$AINdx.$$

The proportionality to N will hold if the space distribution of the grains is comparatively sparse. Again, if ω be the average effective area of a grain and κ a numerical factor, the direct light screened off or absorbed by a single grain can be written $\kappa\omega I$, and that absorbed within the stratum dx,

$$\kappa\omega INdx.^{5}$$

Thus the change of the direct light intensity in passing from x to $x+dx$ will be

$$dI = -(A+\kappa\omega)INdx. \tag{3}$$

Since the developed silver grains are known to show no trace of orientation, the conditions of incidence upon them are the same for direct as for scattered light. Thus the same coefficient $\kappa\omega$ will serve to express the absorption of both, and the energy of scattered light absorbed within the stratum dx will be

$$\kappa\omega I_s Ndx,$$

where I_s has been written for the value corresponding to the depth x. We have, therefore,

$$dI_s = (AI-\kappa\omega I_s)Ndx. \tag{4}$$

This and (3) are the required two equations for I and I_s. The space-density of the grains may vary with the depth. N being any given function of x, introduce as a new variable $n=\int_0^x Ndx$, i.e. the number of grains per unit area, from the surface down to the depth x. Then the equations will become

$$\frac{dI}{dn} = -(A+\kappa\omega)I, \tag{3'}$$

$$\frac{dI_s}{dn} = AI-\kappa\omega I_s, \tag{4'}$$

[5] It can be shown that the treatment based on this expression is ultimately equivalent to one which takes a full account of the effect of overlapping of grains piled above each other haphazardly in different strata dx.

with the conditions $I=I_0, I_s=0$ for $x=0$, i.e. $n=0$. The values of I and I_s to be substituted in the definitions (1) and (2) of the densities will of course be those which follow from (3′), (4′) for $x=$ thickness l of the layer or $n=$ total number of grains per unit area. If the particles were very small compared with the wave length λ, then of the total scattered light only about one-half would be sent in the direction of the incident light and under acute angles θ to it, since in that case the distribution of scattered light is almost symmetrical with respect to the plane $\theta=90°$.[6] For particles, however, whose size is of the order of λ the scattered light can be expected to be largely confined to the anterior hemisphere, $\theta=0$ to $\pi/2$.[7] Thus in our case it can be assumed that the flux I_s obtained from (3′), (4′) for $x=l$ emerges almost entirely on the side of the recording apparatus.

The sum of these two equations gives

$$\frac{d}{dn}(I+I_s)=-\kappa\omega(I+I_s), \qquad \ln\frac{I_0}{I+I_s}=\kappa\omega n,$$

so that, by (1), we have, first of all, the diffuse density

$$D'=M\kappa\omega n, \tag{5}$$

where n is the total number of grains per unit area and $M=\log e\doteq 0.4343$.

This simple form of the diffuse density is a consequence of the extreme assumption that the whole light scattered by each grain is sent forward. If, more generally, only a fraction θ is sent forward and the remainder backwards, then, as will be shown in a subsequent paper, the diffuse density is

$$D'=\log\frac{(a+b)e^{bn}-(a-b)e^{-bn}}{2b},$$

[6] Thus it comes that in Schuster's treatment of the problem of a foggy atmosphere (Astrophys. Journal, *21*, p. 1; 1905) the scattered light is, at each stage, assumed to be sent in equal amounts backward and forward.

[7] Cf. a paper by W. Shoulejkin, Phil. Mag., *47*, p. 307; 1924, based on G. Mie's general form of the solution for a spherical scattering particle. This forward tendency of scattered light with increasing size of scatterer is already very marked in the case of silver particles of diameter 153 $\mu\mu$ for $\lambda=525$ $\mu\mu$, as found by R. Gans (Ann. der Physik, *76*, p. 29; 1925) theoretically, by Mie's formula, and also verified by him experimentally. A much more marked concentration of the scattered radiation in the anterior hemisphere is found for gold particles of 160 and 180 $\mu\mu$, with $\lambda=550$, as investigated in Mie's own paper, Ann. der Physik, *25*, p. 429; 1908. In the latter case almost the whole scattered light is sent forward. Although the optical constants of the spongelike silver grains of the photographic plate will, no doubt, differ from those of metallic silver and gold, the said tendency can be supposed to hold essentially also for these grains. This, moreover, is borne out by the fact, that in our case a direct experimental estimate gave for the total light thrown back, including that reflected at the surface, only 4 per cent of the incident light.

where $a=(1-\theta)A+\kappa\omega$, $b=[\kappa^2\omega^2+2(1-\theta)A\kappa\omega]^{1/2}$. The sub-case $\theta=1$, however, which leads to formula (5), has turned out to cover the observed diffuse densities or at least their relation to the specular densities with sufficient accuracy, for all photographic emulsions thus far examined. It can therefore be concluded that, so far as this relation is concerned, the fraction θ, if at all, differs but little from unity.

Next, equation (3′) gives

$$I=I_0e^{-pn}, \qquad p=A+\kappa\omega, \tag{6}$$

and since $I+I_s=I_0e^{-\omega n}$, we have

$$I_s=I_0(e^{-\omega n}-e^{-pn}). \tag{7}$$

The values (6) and (7) of the emergent light fluxes, introduced into (2), give for the specular density the expression

$$D=-\log\,[\epsilon e^{-\omega n}+(1-\epsilon)e^{-pn}]. \tag{8}$$

Eliminating n by means of (5) and putting $\beta=1+A/\kappa\omega$ we have ultimately

$$10^{-D}=\epsilon\cdot 10^{-D'}+(1-\epsilon)\cdot 10^{-\beta D'} \tag{9}$$

which is the required *relation between the specular and the diffuse densities.* Equivalently we may write, in terms of the corresponding transparencies T and T',

$$T=\epsilon T'+(1-\epsilon)T'^{\beta}.$$

Of the two constants appearing in this relation, ϵ is essentially a positive fraction, and $\beta>1$. In the limiting case $\epsilon=0$, when there is no scattered light in the normally emergent beam, the relation (9) reduces to a mere proportionality, $D=\beta D'$. (An example of this kind is afforded by one of the emulsions to be considered presently.) If the scattering is small compared with the absorption ($A/\kappa\omega$ small), then $\beta\doteq 1$ and the two densities coincide. In general, however, they differ from each other considerably and their ratio deviates perceptibly from constancy, so that the full formula (9) comes to its rights.

In order to determine the numerical values of the two constants ϵ and β from two observed pairs of corresponding densities D, D', the following method can be used.

For small densities equation (9) becomes

$$(\beta-1)\zeta=\frac{D}{D'}-1,$$

where $\zeta = 1-\epsilon$. Thus from an observed pair of small densities D, D' we can determine $(\beta-1)\zeta = a$, say. Formula (9) can now be written, for any densities,

$$\log\left[1-\frac{1-10^{D'-D}}{\zeta}\right] = -(\beta-1)D' = -\frac{aD'}{\zeta}$$

which, with another pair of observed values substituted for D, D', is an equation for ζ alone. This is of the form

$$\log(1-z) = -bz, \qquad z = c/\zeta,$$

where b, c are known numbers. A root z, correct to three figures, say, is readily found by successive approximations;[3] whence follows ζ and thus also $\epsilon = 1-\zeta$ and $\beta = 1+a/\zeta$.

This method has been actually applied in some of the cases to be quoted presently; in the remaining ones the constants ϵ, β were determined by trial and error so as to suit the majority of observations.

Formula (9) has thus far been tested on six sets of data obtained with five different emulsions. For each measured diffuse density D' the corresponding specular density D was calculated by means of this formula and compared with the observed specular density. The agreement, as will be seen presently, turned out to be close enough throughout the sets, the deviations in no case exceeding seriously the limits of experimental error.

Before quoting the results of this comparison some remarks concerning the experimental method may be inserted here.

The data included in tables 1, 2, 3 were obtained by a visual method using two instruments, one for the specular and one for the diffuse density. These instruments were described in a paper by one of us (J.O.S.A. & R.S.I., loc. cit.). Certain optical data concerning these instruments were omitted from the description. These factors may be significant in this and future investigations and it is advisable to state that, in the measurement of specular density with the motion picture densitometer: 1. The angle subtended at the sample by the dividing line of the photometric field of the Martens photometer instrument was equal to about 8′; 2. The dominant wave length of the light used was approximately 5200A.

For the remainder of the data (tables 4, 5, and 6) a thermoelectric photometer newly designed for the investigation of the problem of

[3] Thus, e.g., in the case of $\log(1-z) = -1.401z$, corresponding to the emulsion Eastman 40, the successive approximations, starting from $z=0.9$, were .945, .9525, .9536, .9539, .9539, converging very rapidly, indeed.

light scattering by photographic materials was used. This instrument will be described in detail in a subsequent communication, and only its salient features will be mentioned in this connection. It provides means for the variation of the solid angle subtended by the receiving element, for the variation of the degree of collimation, and for the variation of the dominant wave length of the incident light. Further, the instrument is so constructed that the identical areas employed in the measurement of specular density may be used without change for the measurement of the diffuse density. As an accessory device, a thermoelectric goniophotometer is provided to make possible a point-to-point measurement of angular distribution of transmitted intensity. Results obtained by this device will be used in future investigations.

The specular density values given in tables 4, 5, 6 were measured with the thermo-element subtending an angle of about 1.5° at the sample, and the diffuse densities were measured with the sample placed over an opening in the surface of an integrating sphere. In both cases a cylindrical beam was used to illuminate the sample. The dominant wave length affecting the thermopile was estimated to be of the order of 2μ. The light source used was a tungsten lamp operated at a color temperature of $2760°K$. In each of the following tables the first row contains the observed values of the diffuse density, the second gives the corresponding values of specular density calculated by means of formula (9) with the values of the constants ϵ and β given at the top of each table, and the third row contains the observed values of specular density. It will be seen that the deviations are small.

TABLE 1. *Motion Picture Positive*
$\epsilon=0.250, \quad \beta=1.49_6$

D'_{obs}	0.048	.105	.230	.385	.438	.485	.565	.689	.770	.956	.982	1.50
D_{calc}	0.06_8	$.14_4$	$.31_2$	$.52_0$	$.59_0$	$.65_2$	$.75_7$	$.91_8$	1.02	1.26	1.29	1.91
D_{obs}	0.07_0	$.15_2$	$.31_6$	$.52_3$	$.59_0$	$.65_1$	$.76_1$	$.90_2$	1.01	1.23	1.27	1.95
ΔD	$-.00_4$	$-.00_8$	$-.00_4$	$-.00_3$	$.00_0$	$+.00_1$	$-.00_4$	$+.01_6$	$+.01$	$+.03$	$+.02$	$-.04$

TABLE 2. *Par Speed Motion Picture Negative*
$\epsilon=0.350, \quad \beta=1.85$

D'_{obs}	0.064	.110	.223	.355	.460	.550	.640	.790	1.08	1.31	1.58	1.89
D_{calc}	0.09_8	$.16_8$	$.33_7$	$.52_4$	$.67_5$	$.79_3$	$.91_1$	1.10	1.45	1.71	2.00	2.33
D_{obs}	0.10_8	$.18_9$	$.33_2$	$.50_8$	$.65_0$	$.77_0$	$.89_1$	1.07	1.44	1.75	2.00	2.40
ΔD	$-.01_0$	$-.01_2$	$+.00_5$	$+.01_7$	$+.02_5$	$+.02_3$	$+.02_0$	$+.03$	$+.01$	$-.04$	$.00$	$-.07$

TABLE 3. *Eastman 40*
$\epsilon=0.490, \quad \beta=2.07_8$

D'_{obs}	0.065	.113	.230	.553	.940	1.11	1.24	1.35	1.41	1.65
D_{calc}	0.09_9	$.17_1$	$.33_9$	$.76_2$	1.21	1.39	1.53	1.64	1.71	1.95
D_{obs}	0.10_1	$.17_5$	$.33_2$	$.74_0$	1.23	1.40	1.53	1.61	1.75	1.95
ΔD	$-.00_2$	$-.00_4$	$+.00_7$	$+.02_1$	−.02	−.01	.00	+.03	−.04	.00

TABLE 4. *Velox Emulsion coated on glass*
$\epsilon=0.210, \quad \beta=1.20_0$

D'_{obs}	0.204	.245	.519	.778	1.00	1.30 ± .02	1.38 ± .05
D_{calc}	0.23_8	$.28_9$	$.59_9$	$.89_6$	1.15	1.49 ± .02	1.58 ± .06
D_{obs}	0.22_2	$.29_4$	$.59_8$	$.91_8$	1.17	1.52 ± .01	1.67 ± .02
ΔD	$+.01_4$	$-.00_5$	$+.00_2$	$-.02_2$	−.02		

TABLE 5. *Superspeed Portrait Film*
$\epsilon=0, \quad \beta=1.47_4$, i.e. $D=\beta D'$

D'_{obs}	0.180	.206	.350	.544	.641	.872	.916	1.16	1.32
D_{calc}	0.26_5	$.30_4$	$.51_6$	$.80_2$	$.94_5$	1.28_5	1.35	1.71	1.95
D_{obs}	0.23_5	$.28_9$	$.50_0$	$.79_4$	$.93_9$	1.27	1.35	1.72	1.97
ΔD	$+.02_9$	$+.01_5$	$+.01_6$	$+.00_5$	$+.01_5$	$+.01_5$	.00	−.01	−.02

TABLE 6. *Motion Picture Positive (New measurements, by thermoelectric method.)*
$\epsilon=0.250, \quad \beta=.149_6$

D'_{obs}	0.052	.100	.140	.478	1.01*	1.36
D_{calc}	0.07_1	$.13_7$	$.19_1$	$.64_3$	1.32	1.75
D_{obs}	0.07_2	$.13_3$	$.19_2$	$.65_8$	1.31	1.77
ΔD	$-.00_1$	$+.00_4$	$-.00_1$	$-.01_5$	+.01	−.02

It is worth while to note that the two sets of observations on Motion Picture Positive (Tables 1 and 6), though obtained on different samples of the material and with different apparatus, are excellently represented by the formula (9) with the same values of the constants ϵ, β.

It will be remembered that the constant ϵ expresses the fraction of the total scattered light I_s which is present in the beam transmitted normally or almost so. Now, with increasing size of the particles the scattered light can be expected to be more and more concentrated towards the normal direction (pointing forward). Accordingly one

would expect the value of ϵ to increase with the grain size. Now, for four of the five emulsions used the values of ϵ are, in ascending order,

$$0.210, \quad 0.250, \quad 0.350, \quad 0.490,$$

and this is, in fact, also the order of increasing grain size of these emulsions, viz. Velox, Motion Picture Positive, Par Speed Motion Picture Negative, and Eastman 40. (Such also happens to be the ascending order of the β-values and therefore of the ratios of the scattering to the absorption coefficient, $A/\kappa\omega$.) An exception to this correspondence is the behaviour of Superspeed Portrait Film which shows an almost perfect proportionality of D to D' and thus calls for $\epsilon=0$, although the average grain size of this emulsion is probably greater than that of any of the others.

EASTMAN KODAK RESEARCH LABORATORY,
ROCHESTER, N.Y.
MARCH, 1927.

UNIDIRECTIONAL QUANTA IN WAVE MECHANICS

By G. Breit

ABSTRACT

The problem of emission by an atom is treated according to Schroedinger's rules. The translational motion of the atom as a whole is taken into account. The charge-current density vector is shown to be propagating with the velocity of light in the direction of the emitted quantum. Only unidirectional quanta can be emitted according to the theory. The reason is that the charge-current wave radiates infinitely more intensely if it travels with light velocity than otherwise.

It is shown that Gordon's treatment of the Compton effect involves an analogous phenomenon. It is suggested that Schroedinger's equations designed for the conservation of energy and momentum work out better when the translational motion of the atom as a whole is considered.

A discussion of the unidirectional nature of quanta has been given by Dirac[1] from the point of view of the new mechanics. It seems desirable, however, to have a clear understanding of the mechanism of emission which makes it necessary for quanta to escape unidirectionally. The articles of Gordon[2] and Schroedinger[3] on the Compton effect are very helpful in this connection. The treatment below is given along the lines of Schroedinger. The particular point of interest is that the distribution of charge responsible for the emission is represented by a plane wave having the direction of the emitted quantum for its wave normal and having the frequency and wave length of the quantum. Using a consideration somewhat analogous to Gordon's, it is shown that this charge wave may be thought of as guiding the light wave. The wave of charge is analogous to that introduced by Schroedinger for the Compton effect. The essential difference is that for the Compton effect Schroedinger looks at the wave of charge as a reflecting mirror which acts selectively on account of interference conditions while below the charge wave performs the emission. Simultaneously with a wave of charge density there is also emitted a wave of current density. It is of interest to note that the three components of current density together with the charge density form a four vector which may be interpreted to within a scale factor as the electromagnetic potential of a light wave. The light wave so defined is a plane wave and the quanta emitted move normally to the wave surface. In a sense then the emitted light may be interpreted as the $\psi\bar{\psi}$ wave itself. Such a point of view

[1] P. A. M. Dirac, Proc. Roy. Soc.; *111*, p. 405; 1926.
[2] W. Gordon, ZS. f. Phys.; *40*, p. 117; 1926.
[3] E. Schroedinger, Ann. d. Physik; *82*, p. 257, 1927.

may be of interest in connection with the interpretation of radiation. For simplicity, relativity is neglected and an atom consisting of a single nucleus and a single electron is considered. The Cartesian coordinates and the masses of the nucleus and electron are called respectively $(X_1, X_2, X_3)M$ and $(x_1, x_2, x_3)m$. The potential energy is $V(x_1-X_1, \cdots)$. The differential equation associated with the system is

$$\left\{ \sum_{k=1}^{3} \left(\frac{1}{m} \frac{\partial^2}{\partial x_k^2} + \frac{1}{M} \frac{\partial^2}{\partial X_k^2} \right) + \frac{8\pi^2}{h^2} (E-V) \right\} \psi = 0 \tag{1}$$

We define the relative coordinates y and the center of mass coordinates Y as

$$y_k = x_k - X_k, \quad (m+M)Y_k = MX_k + mx_k \tag{2}$$

and we let

$$m'^{-1} = m^{-1} + M^{-1}, \quad M' = m+M$$

In this notation (2) becomes

$$\left\{ \sum \left(\frac{1}{M'} \frac{\partial^2}{\partial Y_k^2} + \frac{1}{m'} \frac{\partial^2}{\partial y_k^2} \right) + \frac{8\pi^2}{h^2} (E-V(y)) \right\} \psi = 0 \tag{3}$$

Here the Y are separated from the y and presupposing a knowledge of the solutions of

$$\left\{ \frac{1}{m'} \sum \frac{\partial^2}{\partial y_k^2} + \frac{8\pi^2}{h^2} (E-V) \right\} \psi = 0 \tag{3'}$$

solutions of (3) may be written as

$$\begin{aligned} \psi &= \sum C(P,n) \left(exp \frac{2\pi i}{h} [(\mathcal{E}+E_n)t - (PY)] \right) u_n(y) \qquad (4) \\ &= \sum C(P,n) e^{i\varphi} u_n(y) \end{aligned}$$

where

$$(PY) = \sum P_k Y_k, \quad \sum P_k^2 = 2M'\mathcal{E} \tag{5}$$

and

$$\varphi = \frac{2\pi}{h} [(\mathcal{E}+E_n)t - (PY)]$$

The numbers P_k are here arbitrary. The functions $u_n(y)$ and the numbers E_n are the characteristic functions and values of E in (3'). The state of excitation is therefore determined by n while the state of

translation is fixed by the P_k. The kinetic energy of translation is $\mathcal{E}$ and the momentum of the atom as a whole is (P_1, P_2, P_3).

If only two states (P, n), (P', n') are considered then

$$\psi = Ce^{i\varphi}u_n(y) + C'e^{i\varphi'}u_{n'}(y) \tag{6}$$

According to Schroedinger[4] the quantities

$$e_a\int \psi\bar{\psi}\,dx', \quad \frac{he_a}{4\pi i m_a}\int (\psi\,\mathrm{grad}_a\,\bar{\psi} - \bar{\psi}\,\mathrm{grad}_a\,\psi)dx'$$

may be interpreted as the contributions to the charge and current densities at a point due to a charged particle having coordinates (x_a, y_a, z_a) provided ψ is properly normalized and provided dx' and the integrals are taken with fixed x_a, y_a, z_a.

Denoting the charge on the nucleus by e and that on the electron by $-e$ and adding their contributions the current density components are

$$J_k = \frac{he}{4\pi i}\left\{-\frac{1}{M}\int\left(\bar{\psi}\frac{\partial\psi}{\partial X_k} - \psi\frac{\partial\bar{\psi}}{\partial X_k}\right)dx + \frac{1}{m}\int\left(\bar{\psi}\frac{\partial\psi}{\partial x_k} - \psi\frac{\partial\bar{\psi}}{\partial x_k}\right)dX\right\} \tag{7}$$

while the charge density is

$$\rho = e\left\{\int \psi\bar{\psi}\,dx - \int \psi\bar{\psi}dX\right\} \tag{8}$$

Here dx means dx_1, dx_2, dx_3. We have by (2)

$$\frac{\partial\psi}{\partial X_1} = \frac{M}{M'}\frac{\partial\psi}{\partial Y_1} - \frac{\partial\psi}{\partial y_1}, \quad \frac{\partial\psi}{\partial x_1}\frac{m}{M'}\frac{\partial\psi}{\partial Y_1} + \frac{\partial\psi}{\partial y_1}$$

which turns (7) into

$$\frac{4\pi i}{he}J_k = \frac{1}{M'}\left\{\int\left(\bar{\psi}\frac{\partial\psi}{\partial Y_k} - \psi\frac{\partial\bar{\psi}}{\partial Y_k}\right)dX - \int\left(\bar{\psi}\frac{\partial\psi}{\partial Y_k} - \psi\frac{\partial\bar{\psi}}{\partial Y_k}\right)dx\right\}$$

$$+\frac{1}{M}\int\left(\bar{\psi}\frac{\partial\psi}{\partial y_k} - \psi\frac{\partial\bar{\psi}}{\partial y_k}\right)dx + \frac{1}{M}\int\left(\bar{\psi}\frac{\partial\psi}{\partial y_k} - \psi\frac{\partial\bar{\psi}}{\partial y_k}\right)dx \tag{9}$$

In (8) and (9) we substitute (6) by calculating the combinations

$$\psi\bar{\psi} = C\bar{C}u_n^2 + C'\bar{C}'u_{n'}^2 + (C\bar{C}'e^{i(\varphi-\varphi')} + \bar{C}C'e^{i(\varphi'-\varphi)})u_n u_{n'}$$

[4] E. Schroedinger, Ann. d. Physik, *81*, p. 109; 1926.

$$\bar{\psi}\frac{\partial\psi}{\partial Y_k}-\psi\frac{\partial\bar{\psi}}{\partial Y_k}=-\frac{2\pi i}{h}\{2C\overline{C}P_k u_n^2+2C'\overline{C}'P_k' u_{n'}^2+(P_k+P_k')$$
$$(\overline{C}C'e^{i(\varphi'-\varphi)}+C\overline{C}'e^{i(\varphi-\varphi')})u_n u_{n'}\}$$

$$\bar{\psi}\frac{\partial\psi}{\partial y_k}-\psi\frac{\partial\bar{\psi}}{\partial y_k}=\left(u_{n'}\frac{\partial u_n}{\partial y_k}-u_n\frac{\partial u_{n'}}{\partial y_k}\right)(C\overline{C}'e^{i(\varphi-\varphi')}-\overline{C}C'e^{i(\varphi'-\varphi)}) \quad (10)$$

When these expressions (10) are substituted into (8) and (9) the following two typical combinations of integrals result

$$\frac{1}{m}\int g(y)G(Y)dx+\frac{1}{M}\int g(y)G(Y)dX=A$$
$$\int g(y)G(Y)dx-\int g(y)G(Y)dX=B \quad (11)$$

Here $G(Y)$ involves $e^{i(\phi-\phi')}$. (The meaning of $\int f(x,X)dx$ is $\int_{-\infty}^{+\infty}f(z, x^0)dx=\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}f(z_1, z_2, z_3; x_1^0, x_2^0, x_3^0)dz_1, dz_2, dz_3)$. Remembering (2) and applying obvious transformations

$$A=\int g(z)\left\{\frac{1}{m}G\left(x^0+\frac{mz}{M+m}\right)+\frac{1}{M}G\left(x^0-\frac{Mz}{M+m}\right)\right\}dz$$
$$B=\int g(z)\left\{G\left(x^0+\frac{mz}{M+m}\right)-G\left(x^0-\frac{Mz}{M+m}\right)\right\}dz \quad (12)$$

The function y involves the u_n while G as mentioned before involves the $e^{i(\phi-\phi')}$. In order to simplify the expressions we suppose that $G(z)$ varies slowly as compared with $g(z)$, i.e., that the wave length of the wave represented by $e^{i(\phi-\phi')}$ is long as compared with atomic dimensions. Since we are going to find that the wavelength of $e^{i(\phi-\phi')}$ is the wave length of the emitted light this assumption is justified *a posteriori*. However, the assumption is not necessary in order to prove the unidirectional character of emission and the equality of the wave lengths of G and of the emitted light. With this supposition in mind

$$A=\frac{G(x_0)}{m'}\int g(z)dz\ ;\quad B=\int\sum z_k\left(\frac{\partial G(Y)}{\partial Y_k}\right)_{Y=x^0}g(z)dz \quad (13)$$

Both integrals are accurate to within terms in $\partial^2 G/\partial Y_k^2$.

Using (10), (11), (13) equations (8), (9) become

$$\rho=\frac{2\pi e i}{h}\left(C\overline{C}'e^{i(\varphi-\varphi')}-\overline{C}C'e^{i(\varphi'-\varphi)}\right)_{x^0}\int u_n(z)u_{n'}(z)\sum_{k=1}^{3}z_k(P_k'-P_k)dz$$

$$\frac{4\pi i}{he}J_k=\left(C\overline{C}'e^{i(\varphi-\varphi')}-\overline{C}C'e^{i(\varphi'-\varphi)}\right)_{x^0}\left\{-\frac{4\pi^2}{M'h^2}(P_k+P_k')\times\right.$$

$$\int(\sum z_k(P_k'-P_k)u_nu_{n'}dz$$

$$\left.+\frac{1}{m'}\int\left(u_{n'}\frac{\partial u_n}{\partial y_k}-u_n\frac{\partial u_{n'}}{\partial y_k}\right)_{y=z}dz\right\} \tag{14}$$

Both the charge and the current density are proportional to the real part of $CC'e^{i(\phi-\phi')}$ and are therefore represented by plane waves with the phase:

$$\varphi-\varphi'=\frac{2\pi}{h}[(\mathcal{E}+E_n-\mathcal{E}'-E_{n'})t-\sum(P_k-P_k')x_k^0] \tag{15}$$

It is clear even without calculation that the phenomena of emission are particularly simple if the velocity of this wave is the velocity of light. Only in that case is the field radiated by the current flow (14) propagated in the direction of the wave itself. The current flow (14) is associated with an electro-magnetic field in its immediate neighborhood. If therefore (15) does not represent the phase of a possible light wave we must have in (14) an electro-magnetic wave moving slower than light and in addition one or more waves moving with light velocity at a proper angle with the vector P_k-P_k'. This is unlikely. It is particularly improbable because we have not taken into account the radiation force $2/3\, e^2/c^3\bar{v}$. This is a common malady of most modern theories. The effect of this force, if it were possible to treat it mathematically, would be presumably that of modifying ρ and J_k. The mechanism of the modification may be imagined as follows: Every portion of space radiates energy in accordance with electromagnetic laws. The light waves so radiated perturb the charge distribution. Only if (15) represents the phase of a possible light wave will the disturbance in (14) leave the frequency and the velocity of propagation of (14) unchanged. Otherwise the light waves from the back portions of the wave will get ahead of the charge wave and will produce in it a disturbance depending on their relative velocities.

We can satisfy ourselves, however, as to these facts also by calculation in a manner analogous to that of Gordon in the case of the Compton

effect though with much less work. In order to calculate the probability of a transition from the state (P, n) to the state (P', n') we must calculate the probability of radiating energy by the distribution (14). The first step is to calculate the retarded potentials. In the calculation of all the quantities we use the retarded phase $[\phi-\phi']$. If the point at which we wish to calculate the potentials is a distance r from the center of the atom it is to a sufficient approximation at a distance $r-\sum\xi_k x_k^0$ from x^0 where ξ^k are the direction cosines of r. The retarded phase is then

$$[\varphi-\varphi']=\frac{2\pi}{h}\left[(\mathcal{E}+E_n-\mathcal{E}'-E_{n'})\left(t-\frac{r}{c}\right)\right.$$
$$\left.-\sum\left(P_k-P_{k'}-\frac{\xi_k}{c}(\mathcal{E}+E_n-\mathcal{E}'-E_{n'})\right)x_k^0\right]$$

Since $e^{i[\phi-\phi']}$ enters into the integrals for the potentials and since the $x^0{}_k$ do not enter in any other way, the value of these integrals is very much (infinitely) greater for

$$P_k=P_{k'}+\frac{\xi_k}{c}(\mathcal{E}+E_n-\mathcal{E}'-E_{n'}) \tag{16}$$

than for any other relation between the P, P'. The meaning of (16) is obvious. The momentum of the atom before emission (P_k) is equal to the momentum of the atom after emission $(P_{k'})$ plus the momentum of the emitted quantum
$\xi_k/c(\mathcal{E}+E_n-\mathcal{E}'-E_{n'})=\xi_k\, h\nu/c$. Using (16) in (15)

$$\varphi-\varphi'=2\pi\nu\left(t-\sum\frac{\xi_k x_k}{c}\right) \tag{17}$$

which is exactly the phase of the quantum.

The above proof is, however, incomplete because we have specified a given direction of emission ξ. We must really show in addition that if the unit vector ξ is made variable then the total radiation is infinitely greater for such $P-P'$ which make

$$P_k-P_{k'}=\frac{\xi_k}{c}\left\{h\nu_0+\frac{1}{2M'}\sum_{l=1}^{3}(P_l^2-P_l'^2)\right\} \tag{18}$$
$$h\nu_0=E_n-E_{n'}$$

This, however, is very easily shown by integrating the expressions (14) through a large sphere and then integrating over the ξ_k. It is also

obvious from the fact that unless there exist direction cosines ξ_k such as to make (18) satisfied no infinite contributions to the integral are made. We see, therefore, that charge waves (14) which are due to transitions satisfying (16) radiate infinitely more energy than charge waves not satisfying this condition. Such transitions are, therefore, the only ones which may occur.

By taking into account the nucleus and its translational motion we have obtained a charge current distribution which exists only if two or more states of the atom exist at the same time. If, therefore, we should follow out Schroedinger's[5] proposed system of equations for the connection between the electromagnetic field and matter contributions to the electromagnetic potentials due to the existing charge distribution disappear unless the atom is in a state of transition. Therefore, the difficulty mentioned by Schroedinger with the hydrogen spectrum would disappear as well. When two states are present simultaneously the current charge wave contributes a field which is of just the right frequency to cause perturbations in the atom leading to quantum jumps. Schroedinger's article just referred to includes, therefore, at least qualitatively, the formalism required for the discussion of spontaneous radiation. An exact treatment of this question must be connected with Slater's.[6]

The picture of the unidirectional character of quanta which has been presented is not a departure from either Schroedinger's or Gordon's point of view. In fact, Schroedinger's reflecting mirror $\psi\bar{\psi}$ wave has a meaning only inasmuch as perturbations in it are induced by the light wave. The character of the perturbations is calculated by Gordon. The second term in his equation (39) gives the essential part. The essential part of the term $e^{i(\delta f+\phi)}$. Here $\delta f+\phi=2\pi/h(p-p'+\pi, x)$. But according to his equation (44) $p-p'+\pi=\pi^*$. Hence $\delta f+\phi=2\pi/h$ (π^*, x) and the charge-current four vector here also propogates with the emitted quantum.

DEPARTMENT OF TERRESTRIAL MAGNETISM,
CARNEGIE INSTITUTION OF WASHINGTON,
MARCH 14, 1927.

[5] E. Schroedinger, Ann. d. Physik, *82*, p. 265; 1927.
[6] J. C. Slater, Proc. Nat. Acad. Sci., *13*, p. 7; 1927.

INSTRUMENT SECTION

EDITORIAL

For several years, the Association of Apparatus Makers of the United States of America, in order to encourage the publication of papers pertaining to new instruments and new methods of measurement, has offered a prize of $250 for the best paper appearing in any given calendar year in this JOURNAL. The prize for the year 1926 has been awarded to Professor Horatio B. Williams, Dalton Professor of Physiology at the College of Physicians and Surgeons, New York City, for his paper on, "The Einthoven String Galvanometer," which appeared in the September 1926 issue. This paper by Dr. Williams was the second installment of a series of papers on the Einthoven type of instrument, the first part having appeared in August 1924. Honorable mention was made of the paper by Mr. Leo Behr on, "A New Relative Humidity Recorder," which appeared in the June 1926 issue. The committee of award for 1926 consisted of Professor S. R. Williams of Amherst College, Chairman; Dr. Albert W. Hull of the General Electric Research Laboratory at Schenectady and Professor W. G. Cady of Wesleyan University, Middletown, Conn.

The Association of Scientific Apparatus Makers has offered a similar prize for the best paper on instruments, other than optical, appearing in this JOURNAL during the calendar year 1927. Contributors sending in manuscript for publication need not request that the paper be considered for the award, since all eligible papers will be automatically considered by the committee.

In awarding the prize due weight will be given to each of the following:

(1) NOVELTY AND ORIGINALITY. It is essential that the paper should describe new and original work by the author or authors. Reviews or Summaries are not to be considered.

(2) SCIENTIFIC VALUE. The importance of the paper as a contribution to research or instruction will be given special consideration.

(3) EXPOSITION. Articles will also be judged on clearness, conciseness, and general arrangement of material. Special attention will be paid to diagrams. While brevity is a virtue in scientific writing, brevity ceases to be a virtue when it is carried to the point of introducing obscurity or of omitting essential details.

No articles can be considered as entered for the prize which have been accepted for publication elsewhere, unless it is arranged that such publication shall be subsequent to appearance in this Journal.

A similar prize for the best paper on instruments submitted for publication during the calendar year 1924 was awarded to Mr. Loyd A. Jones of the Research Laboratory of the Eastman Kodak Company for his paper on, "A New Method for Photographic Spectrophotometry," which appeared in the May 1925 issue.

The editors of the JOURNAL wish to take this opportunity of expressing their appreciation to the Association of Scientific Apparatus Makers of the United States of America for their generosity in offering this prize and for the whole hearted way in which the Association has left to the JOURNAL all specifications with regard to the way in which the prize shall be awarded.

Magnetostriction of single crystals of iron.—The experiments of Webster were performed with rods 18 mms long, with their axes parallel to the tetragonal or trigonal or digonal axes of the atom-lattice; those of Honda and Mashiyama with rods 40 mm long having their axes oriented at known angles to the crystal axes, and with ellipsoidal discs having their planes of symmetry normal to the tetragonal or trigonal or digonal axes of the lattice. Webster's data show that the longitudinally-magnetized rod expands, if its axis is a tetragonal axis of the lattice; contracts, if its axis is a trigonal one; and undergoes expansion followed by contraction (like polycrystalline soft iron) if its axis is parallel to the digonal axis of the lattice. The expansion along the tetragonal axis attains values as high as 20 parts in a million (four or five times as great as in polycrystalline metals) when the magnetization approaches saturation. The curves of change-in-length vs. magnetization show no signs of bending over into parallelism with the axis of magnetization as saturation is approached, though in these experiments it was never quite reached. Tensile stress of the order of a few kg/mm^2 increases the permeability for fields parallel to the tetragonal axis; decreases it, for fields parallel to the trigonal axis; while for fields applied along the digonal axis there is a Villari reversal, as in polycrystalline iron. The data of Honda and Mashiyama are in general agreement with those of Webster, and are completed by an extensive series of measurements on the transverse expansions and contractions associated with longitudinal magnetizations. [W. L. Webster, (Cavendish); Proc. Roy. Soc., *A109*, pp. 570–584; 1925. K. Honda and Y. Mashiyama (Tokyo); Tohoku University Sci. Rep., *15*, pp. 755–776; 1926.

KARL K. DARROW

SOME EXPERIMENTAL METHODS IN MAGNETOSTRICTION*

By S. R. Williams

In the classification of magnetic phenomena those which come under the head of magnetostriction form a very large group. It is that set of phenomena in which there are changes in dimensions attending magnetization and conversely the changes in magnetic properties accompanying mechanical stresses. These inter-related phenomena seem to be special cases of a more general principle: *there is one and only one set of physical and chemical properties corresponding to a given set of magnetic characteristics, and conversely there is one and only one set of magnetic properties corresponding to a given set of physical and chemical characteristics.* This means to say that if a magnetic field increases the length of a ferromagnetic rod then a mechanical pull when applied to the same specimen will increase its permeability beyond that of the unstretched condition. It is one of the very significant features of magnetostriction that there is a reciprocal relation[1] which always exists between the deformation caused by magnetization and the change in magnetic property due to a deforming force. Many of the reciprocal magnetostrictive effects are illustrative of the Principle of Le Chatelier, which says that any system whether in physical or chemical equilibrium, if disturbed by some external stress will automatically tend to alter so as to undo the effect of the stress.

The phenomena of magnetostriction have a two-fold significance, one for pure and the other for applied science. Its importance for the field of pure science is that it must eventually yield a more comprehensive theory of atomic structure. It is a safe assertion to make that when the elementary magnet has been discovered a much better concept of the atom will be forthcoming. The atomic structure must have something to do with the phenomena of magnetostriction and therefore the latter may be expected to yield some supplemental conception of the architectural design of the atom.

From the practical standpoint, magnetostriction has a contribution to make, because it seems quite probable from what has already been done, that it will be possible to study the mechanical properties of steel,

* One of a series of reports prepared at the request of the Committee on Research Methods and Technique appointed by the Division of Physical Sciences of the National Research Council. See Editorial in this Journal, *9*, p. 410; 1924.

nickel, cobalt and other ferromagnetic substances by their magnetic behavior.[2] If from ordinary magnetic properties it is possible to arrive at mechanical properties, surely the magnetostrictive effects in which magnetic and mechanical properties always play such an intimately associated role must contribute greatly to our knowledge of magneto-mechanical analysis. This phase of the subject would be of immense value to the steel industry where it has already been shown that magnetic analysis is quite dependable for picking up deep seated flaws. There are two methods at present, practically the only two, for testing the homogeneity of a substance without destroying the specimen; these are the magnetic and x-ray methods[3] of analysis. To be able to detect the presence or absence of flaws in a steel member of any structure in situ or before being set in place or before being machined would be a contribution of the first order. Such a proposition does not seem impossible.

If the subject of magnetism is important and in particular that section of magnetic phenomena devoted to magnetostriction, then the method and technique of research in this field should be set forth and attention called to the numerous problems which remain unsolved.

There are still many problems which might be undertaken with comparatively simple and inexpensive apparatus and the subject of magnetostriction could be studied in many laboratories where today no research is being done. *It is not a question of "What could I do if I had the equipment" but "What can I do with the equipment at my disposal."*

There are several outstanding needs in all research work in magnetism. 1. An improvement in all forms of instruments for magnetic measurements. This in itself constitutes an important field of research. 2. Repetition, of most of the work which has already been done, using improved methods and materials whose previous history is known. Modern methods of pyrometry, microphotography, chemical and x-ray analysis give no excuse for ignorance of the character of the material used in magnetic investigations. 3. The importance of correlating various magnetic phenomena by data obtained from the same specimen cannot be over estimated. 4. Cooperative research is highly desirable. A group of physicists, mineralogists expert in x-ray analysis, chemists and metallurgists cooperating in the study of magnetic phenomena could make a wonderful contribution to this field. 5. Sustained research, both correlative and cooperative is essential to definite advances.

The following outline will present to the eye the main features described under the topic of,

MAGNETOSTRICTIVE EFFECTS

I. Mechanical Strains due to Magnetic Stresses
  (a) Linear Changes
    1. Joule Effect,—Change in length due to longitudinal magnetic field
    2. Transverse Joule Effect,—Change in dimensions normal to magnetic field
    3. Guillemin Effect,—Bending due to a magnetic field
    4. Changes in Young's Modulus
  (b) Circular Changes
    1. Wiedemann Effect,—Twist due to interaction of circular and longitudinal fields
    2. Twisting of rods having a permanent torsional set when magnetized either by a longitudinal or a circular field
    3. Change in the coefficients of rigidity due to a magnetic field
  (c) Volume Changes
    1. Barrett Effect,—Volume change due to magnetization
    2. Changes in bulk modulus

II. Magnetic Strains due to Mechanical Stresses
  (a) Linear Changes
    1. Villari Effect,—Change in induction due to longitudinal stress
    2. Transverse Villari Effect,—Change in induction due to normal stress
    3. Converse of Guillemin Effect,—Change in induction due to bending
  (b) Circular Changes
    1. Second Wiedemann Effect,—Longitudinal magnetization due to twisting a circularly magnetized rod
    2. Wertheim Effect,—Circular magnetization due to twisting longitudinally magnetized rod
    3. Twisting rods with permanent torsional set and obtaining longitudinal or circular fields
  (c) Volume Changes
    1. Nagaoka-Honda Effect,—Change in induction due to hydrostatic pressure

III. Allied Phenomena
    1. Change in resistance due to magnetizing field
    2. Change in Thermo E M F's
    3. E M F's due to magnetization
    4. Change in frequency of a tuning fork when magnetized
    5. "Magnetic Tick"
    6. Barkhausen Effect

I. MECHANICAL STRAINS DUE TO MAGNETIC STRESSES

A. LINEAR CHANGE

1. *Joule effect—Change in length due to a longitudinal magnetic field.* The variations in dimensions produced by a magnetic field are very small and in order to measure them, methods of amplifying these changes must be developed. The Joule effect which is a straight change in length requires some form of an extensometer. Four of them are shown schematically in Figs. 1, 2, 3 and 4 respectively and embrace the main characteristics of the many methods employed. McKeehan and Cioffi, Honda and Mashiyama[4] have recently devised new and

sensitive methods, the intricacies of which may be studied in the original papers.

Joule[5] first discovered the change in length of an iron rod when magnetized and found it to be an increase. His device for measuring the

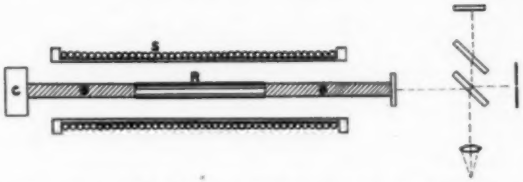

FIG. 1. *The movable mirror of the Michelson Interferometer is attached to the end of the brass extension of the specimen. As the rod contracts or elongates, the interference fringes shift across the field of view. The change in length of the ferromagnetic rod may thus be measured in terms of wave-lengths of light.*

variations in length was a series of two levers in tandem whose motion could be further magnified by a microscope with micrometer eye-piece.

Bidwell[6] extended the work of Joule and showed that in the case of iron the increase in length attained a maximum at a certain field strength, beyond which the rod retracted and finally became shorter than in the unmagnetized condition, Fig. 13. Bidwell worked with toroids and measured the change in diameter as the magnetizing

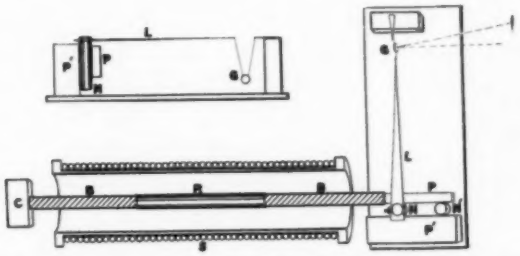

FIG. 2. *When the rod R contracts or elongates, the plate P is pushed along P'. This causes the two steel rollers, N and N', to roll and carry the lever arm L along with them. This deflects the mirror G which may be measured by a telescope and scale.*

current was increased. His amplifying device, Fig. 5, was somewhat on the order of Fig. 4. The development of the Michelson interferometer led to its use in measuring changes in length. This is shown in Fig. 1 and has been used by Lochner[7] and Stevens.[8] Combinations of mechan-

ical and optical levers are shown in Figs. 2 and 3. The first was used by Austin and Guthe[9] and the second was developed by the writer.[10] The unique features of Austin and Guthe's device are the small steel rollers (needles) N, N' between the two glass plates P and P'. As P moves back and forth the needle N rolls and thus deflects the mirror G which is hung bi-filarly. Both forms are adaptable to photographic registration as well as for visual observations. Fig. 4 shows a form used by More[10] and Brackett.[10] Stress is laid on the methods employed in measuring the Joule effect because it is the simplest and most straight forward of all the magnetostrictive effects.

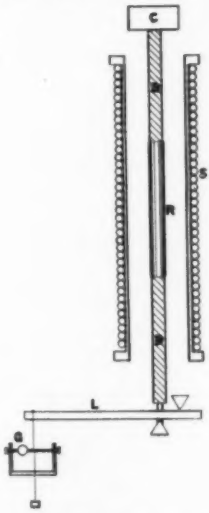

FIG. 3. *As the rod R changes its length, the lever L rises and falls with the changes in length. This causes the mirror, G, to be deflected, because a fine phosphor bronze strip is wound around a roller carrying G and therefore moves with L. The phosphor bronze strip is held taut by a small weight.*

In using the outfit, shown in Fig. 3, for visual observation, the spot of light is thrown on a long strip of paper mounted vertically. As the needle of the ammeter recording the current in the solenoid reaches certain definite divisions on the scale the position of the spot of light on the paper is marked with a pencil and thus in a very short time a whole series of observations may be taken and then measured at one's leisure.

In the Figs. 1, 2, 3 and 4, S is the magnetizing solenoid, R is the ferromagnetic rod under investigation, B, B, its brass extensions, C is the

clamp for the fixed end of the rod, while G is the mirror and L the mechanical lever. The detailed descriptions of these devices are given in the original papers to which references are given.

Due to the exceedingly small variations in dimensions the utmost care must be maintained in order that no outside factors[11] produce changes which would blanket the effects sought for. Particularly temperature changes must be avoided and every means used to maintain a uniform temperature.[12] Uniform magnetic fields must be sought for and means employed to test the magnetic fields used.[13] The specimens to be tested must be placed symmetrically in the field; otherwise

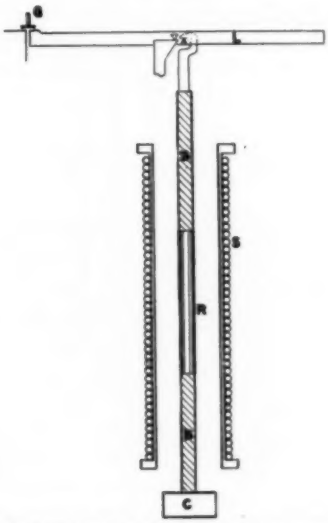

FIG. 4. *Any change in the length of R causes the lever L to move and so tilt the mirror G.*

there will be a mechanical traction on the specimen which will vitiate the results. The specimens should also be mounted on supports which are as independent as possible of those on which the coil producing the magnetic field is placed. The method of supporting the specimens on the coil itself is to be avoided.[14] All extraneous magnetic fields, earth's field, fields of ammeters, etc., must be eliminated. This is particularly true for nickel which is hyper-sensitive to stray fields. The results for nickel are wholly different when the earth's field is present and when counterbalanced by an opposing field. All test samples must be thoroughly demagnetized before a set of readings is taken. The process

consists of putting the sample in an alternating current magnetic field and slowly cutting the a.c. down to zero. It must be done in a space free from all outside fields. If an extraneous field is present when nickel, for instance, is demagnetized the sample will be found magnetized in the direction of the extraneous field. The maximum field of the a.c. with which the demagnetization process begins must be as great or greater than the maximum d.c. field to which the specimen has been originally exposed. To control these magnetic fields a liquid rheostat[16] is to be especially commended. For currents up to 5–7 amperes they work very well where the current is on for a short time.

There are a great many conditions and variations under which the Joule effect may be tested. Joule, Bidwell, Brackett, G. Klingenberg,

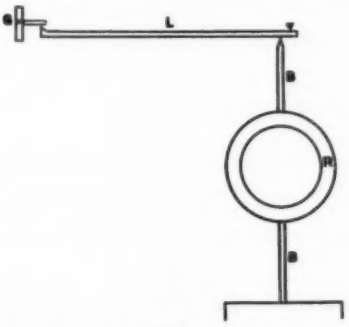

FIG. 5. *Bidwell's method involved the measuring of the change in the diameter of the toroid which he used. Any change in the diameter of R raised or lowered the lever, L, and thereby tipped the mirror G.*

Tangl and most extensively, Honda and Shimizu[17] have studied the way in which changes in length occur when the rods are under tension. Tension produces a profound influence on the Joule effect, even reversing its character under certain conditions. In order to make observations of this kind it is only necessary to arrange for attaching a stretching force to the rod while the Joule effect is being measured.

The Joule effect has been observed under large ranges of temperature, from $-186°$C up to 1020°C. Honda and Shimizu[18] have been investigators in this field.

Just as there is hysteresis in the process of magnetization so is there a hysteresis attending the change of length by magnetization. Knott, Nagaoka and Williams[19] have made observations on these magnetic-elongations and magnetic-twist cycles. The same equipment is needed

for this work as for the regular Joule phenomenon. It is also possible according to McCorkle[20] to study the Joule effect under anhysteretic conditions in which case the initial elongation of iron does not appear.

Because of hysteresis the change in length which accompanies magnetization due to an alternating field must be different from that for a steady field. This is indicated by the work of Honda and Shimizu, Austin and Brown.[21] The close relation between magnetostriction and ordinary hysteresis has been exhibited by Buckley and McKeehan and by Wwedensky and Simanow.[22]

The study of magnetostrictive effects in single crystals[23] is highly interesting and appears to offer a clearer insight into the behavior of rods of the same material where the small crystals which compose the sample are oriented in every direction.

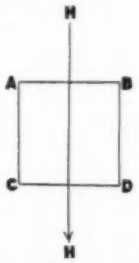

FIG. 6. *Showing the relations between the longitudinal and transverse Joule magnetostrictive effects.*

2. *Transverse Joule effect—Change in dimensions normal to magnetic field.* The first effect discovered by Joule in magnetostriction led him to wonder if a volume change occurred simultaneously with the change in length. Due to lack of sensitivity he was not able to find it and so looked for a change in dimension normal to length of rod to compensate for the change in length. This effect he found. Joule's first scheme[24] was to run an insulated wire through an iron tube and the change in dimensions normal to the field could be observed by measuring the change in length of the tube.

Further investigations of this change in dimension have been made by Bidwell, Williams, Brown and Heaps.[24] There are two ways in which one may compare the Joule transverse with the Joule longitudinal effect. Let *ABCD* be the cross-section of a cube of iron. If a magnetic field *H* is imposed on the cube, Fig. 6, in the direction of the arrow, measurements along *AC* and *CD* would constitute observations for the longi-

tudinal and transverse effects respectively. In this case the direction of the magnetic field is kept constant and the measuring device is shifted. On the other hand if the field, H, is first applied parallel to AC and its change in dimension observed, that would be the longitudinal effect as in the first case, but leaving the measuring device in position the magnetic field may be applied parallel to CD and any changes in length along AC would be a transverse effect. In this second case the field has been shifted instead of the measuring apparatus.

As a rough approximation it might be said that the transverse and longitudinal effects are opposite, i.e. when the elongation occurs longitudinally contraction occurs transversely and vice versa.

Inhomogeneity, and the fact that there is a volume change as well prevents an exact reversal of the two effects.

3. *Guillemin effect—Bending due to a magnetic field.* The literature on this subject is very meagre and for the most part of ancient lineage.

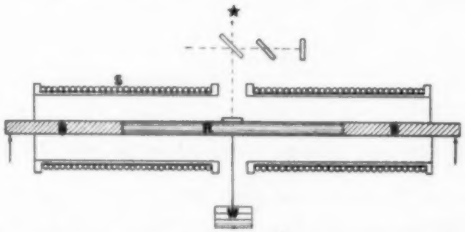

FIG. 7. *Weights, W, were added to bend the rod R. An application of a magnetizing force parallel to the rod caused the interference fringes to shift in the field of view.*

Guillemin[25] observed that if an iron rod, one centimeter in diameter and 20 to 30 cm long, was clamped at one end and placed coaxially in a horizontal solenoid it would tend to straighten when magnetized after being slightly bent by a small weight placed on the free end. Wertheim[26] investigated this same problem and studied especially the effect of the rod not being coaxial with the magnetizing coil. This would naturally make a difference in the effect. Honda, Shimizu and Kusakabe[27] obviated any trouble due to this cause by winding the magnetizing coil directly upon the rod, so that as the rod bent the coil would also. Undoubtedly the best work which has been done on this phase of the subject is contained in this last paper. Stevens and Dorsey[28] have studied the change in elastic constants of steel by this method and obtained positive results. Their method involved a clever application of the Michelson interferometer principle and is shown in Fig. 7. Recently

Miller[29] presented a paper on magnetostriction in which a horizontal wire is anchored at both ends and the sag, when the wire is longitudinally magnetized, is a measure of the change in length of the wire. This is in part a phase of the Guillemin effect. The effect, however, is so involved that it is difficult to separate out the essential features. Guillemin's effect has the unfortunate condition that the rod or wire in a horizontal position has the upper half in a state of tension, while the lower half is compressed. Fig. 8. When the rod or wire shortens or lengthens according to the Joule effect, does this differential effect cause the bending or sagging or is there a tendency for the longitudinal fibers of the bar to set themselves parallel to the field? In the effect of a permanently twisted rod seeking to untwist when longitudinally magnetized it would appear that the second effect must be present. In our present state of knowledge concerning the Guillemin effect it does not seem the most logical way of getting at the general phenomenon which underlies magnetostriction.

FIG. 8. *The upper half of the bar is stretched and the lower half is compressed.*

4. *Changes in Young's modulus.* If one gives equal weight to the earlier and later papers on this subject they would appear very confusing. Wertheim and Tomlinson[30] were very sure that there was no effect of a magnetic field on the modulus of stretch. Brackett[31] found an increase of E amounting to about one-half per cent. In some experiments on the bending of rods due to a magnetic field Stevens and Dorsey[31] found an increase in E both for wrought iron and steel. Later Stevens[32] confirmed this increase of E by stretching experiments which is a more direct method. On the other hand the compression modulus showed no change. The most convincing and thorough going work was done by Honda, Shimizu, and Kusakabe, Honda and Terada,[34] in which they found an increase in the modulus of stretch due to a magnetic field in iron, steel and cobalt. "The elastic constant of a substance in a magnetic field is to be defined as the ratio of the stress applied to the strain caused thereby, the magnetic force constantly acting on the substance. The change of elasticity is then the difference of this quantity when the magnetic field is on and when it is off." Because of hysteresis much of the former work on this subject has no significance because the

right order of applying the field and the stress was not observed. Nickel showed a decrease of E in weak fields and an increase in strong fields. Their work seems to pretty thoroughly establish the positive character of the effect. Their methods for measuring the changes in the modulus was by the ordinary means employed in the laboratory. Further work on this problem should seek to develop greater refinements in the present methods of measuring Young's modulus.

B. CIRCULAR CHANGE

1. *Wiedemann effect—Twist due to interaction of circular and longitudinal fields.* If simultaneously a circular and a longitudinal field are

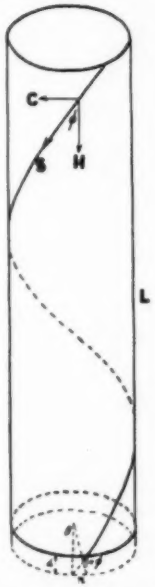

FIG. 9. *Showing the direction of the resultant magnetic field when a circular and a longitudinal magnetic field are superimposed on each other.*

impressed on a rod of iron or other ferromagnetic substance the resultant field is directed along a helix about the rod. Fig. 9. If a change in length occurs along this helix a twist of the rod ensues. This is what Wiedemann[35] discovered, viz., that if an electric current is sent through the rod, producing a circular field, and at the same time it is magnetized longitudinally, a twist in the rod occurs.

The character of it is such that it, qualitatively at least, agrees with the phenomena observed in the Joule effect and makes the effect a special case of Joule's discovery. The difficulty in getting quantitative relations is in ascertaining correctly the distribution of the circular magnetic field. It is somewhat better in the case of the Wiedemann effect in tubes[36] where the circular field is produced by a current through a wire which runs inside of the tube, but is insulated from it. The twist is observed by simply attaching a mirror to the end which is not fixed. Fig. 10. No amplification of the twist is needed, as it may easily be

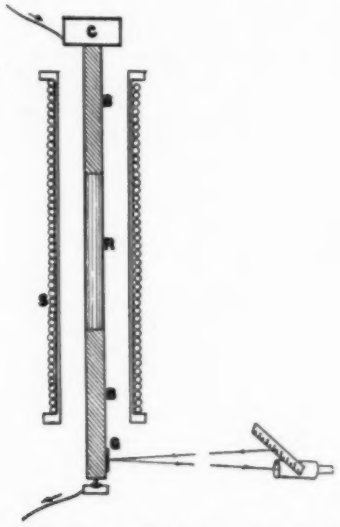

FIG. 10. *The twisting of the rod R due to the interaction of the longitudinal and circular fields may be observed by means of the mirror, telescope and scale.*

measured directly with telescope and scale. The Wiedemann effect may be studied in three different ways. 1. Holding the circular field constant and increasing the longitudinal field from zero upward; 2. Holding the longitudinal field constant and increasing the circular field from zero upward; 3. Varying both fields simultaneously. The author is studying the last condition at the present time. The Wiedemann effect has been the subject of a great many investigations.[37]

2. *Twisting of rods having permanent torsional set when magnetized longitudinally or circularly.* Wiedemann, Smith, Groesser and Williams[38] have observed that if a rod is given a permanent torsional set, a longi-

tudinal magnetic field will cause it to untwist and the direction of the imposed magnetic field has nothing to do with the direction of the untwisting of the rod. It would appear that the filaments of metal along the direction of permanent set tend to straighten out parallel to the imposed field and so untwist the rod. This is a difficult effect to study because it is impossible to obtain homogeneous bars with uniform twist. It needs further study, however, and supposedly the rod with a permanent twist ought to have a torque put upon it when magnetized circularly. These effects may be observed in the same way as the Wiedemann effects.

3. *Change in the coefficient of rigidity due to a magnetic field.* The earlier work on this subject is contradictory. Wiedemann[39] observed that the torsion of an iron wire was diminished by magnetization. Kimball[40] claimed he found an increase in rigidity for iron amounting to 0.9 per cent at the maximum magnetization to which he took the material.

In all of the work on this subject the method was to observe the coefficient of rigidity of the specimen in both the magnetized and the unmagnetized state and obtain the difference. Of all the elastic moduli, the coefficient of rigidity seems to be affected the most by magnetization Among the various investigations, Tomlinson, Barus, Day, Stevens, Honda, Shimizu and Kusakabe[41] there seems to be this agreement that there is an increase in the coefficient of rigidity of iron. The last named workers found that not only iron, but also steel and cobalt increased their rigidity with magnetization while nickel showed a decrease. Honda and Terada[41] have carried out some excellent work on this subject also. In all of the investigations on the changes of the elastic moduli the same criticism seems necessary that not enough refinement was made in the methods for studying the elastic constants.

C. VOLUME CHANGE

1. *Barrett effect—Change in volume due to a magnetic field.* This is the most elusive of all of the changes in dimension which are measured in magnetostriction. The early investigators doubted its presence. Joule[42] looked for a change of volume by placing the piece of iron in a tube which terminated in a capillary, but did not find any effect. This was due in part to the fact that the specimen was not magnetized to saturation. Barrett[43] detected just a slight change in the volume of nickel. His work was qualitative. Bidwell[44] gives Knott[45] credit for first detecting change in volume due to magnetization, but Knott's

work was with hollow iron, steel and nickel tubes in fields that were not uniform and therefore his results are questionable. The best results were obtained by Nagaoka and Honda[46] who worked with a dilatometer containing the specimen in ovoid form which would give uniform distribution of magnetic field. See Fig. 11.

There is a great need for a comprehensive theory covering the problem of deriving a theoretical relationship between the strength of the magnetizing field and the consequent strains of the magnetic medium. Maxwell, Helmhoff, Kirchhoff, Thomson, Larmor, Houston and Sano[47]

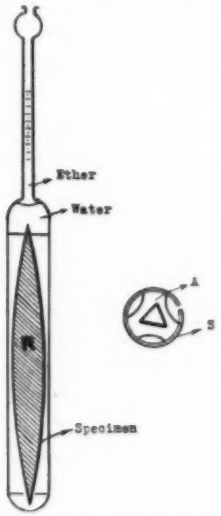

FIG. 11. *As the volume of R changed due to magnetic field, the water surrounding the specimen would be forced up or down the capillary tube and so the change in volume measured. A, S are the rings and supports which held the specimen in place while being magnetized.*

have made attempts at this problem and while a fair qualitative relation has been developed, they all leave much to be desired in the way of quantitative agreement between theory and experiment. This is a field of research needing the attention of the theoretical physicist.

2. *Change in bulk modulus due to a magnetic field.* This is not a well established phenomena as it is extremely difficult to measure, being a very small quantity. The work of Bock[48] bears upon this subject. The evidence for its existence rests upon the presence of the stretch and rigidity modulus and that there is a change in volume due to a magnetic field. The strongest evidence is indirect.

MAGNETIC CHANGES DUE TO MECHANICAL STRESSES

A. LINEAR CHANGES

1. *Villari effect—Change in induction due to longitudinal magnetic field.* Matteucci[49] seems to have been the first to have discovered that pulling or stretching a ferromagnetic rod changes its magnetic induction. Villari[50] however, with greater refinement of method found that if an iron rod is pulled when weakly magnetized it will increase its intensity of magnetization and decrease it when pulled in a strong magnetic field. This is known as the Villari Reversal effect and the point V, Fig. 12, where pulling (or compressing) does not affect the intensity of magnetization is called the Villari reversal point. The Villari effect is just the reciprocal of the Joule effect. Just as the Joule

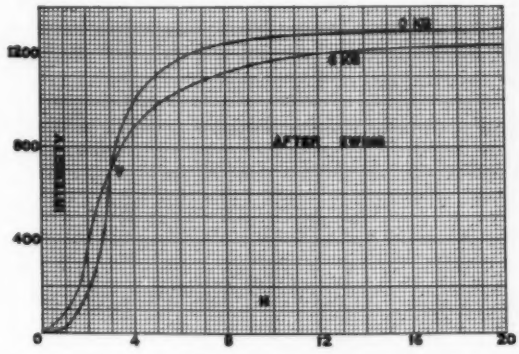

FIG. 12. *Magnetization of annealed iron under various amounts of longitudinal pull.*

effect was different in various substances so also does the Villari effect vary with the sample of material used.

In iron there is first an increase in magnetization and then a decrease as the field is varied from zero and upwards, corresponding to the Joule effect in an iron rod in which the rod will first lengthen and then shorten as the field is increased from zero and upwards. Nickel shortens for all field strengths in the Joule effect and so for the Villari effect there is a decrease in intensity of magnetization when the rod is pulled for all magnetic field strengths thus far studied. Cast cobalt when pulled decreases its magnetization in low fields and increases it for strong fields. Some of the alloys of manganese show a lengthening for all field strengths, suggesting that the four ferromagnetic substances might show the four different types of Joule and Villari effects, Fig. 13. As in

the Joule effect, the previous history of the rods, heat treatment, magnetization, stresses, etc., will to a large extent determine the character of

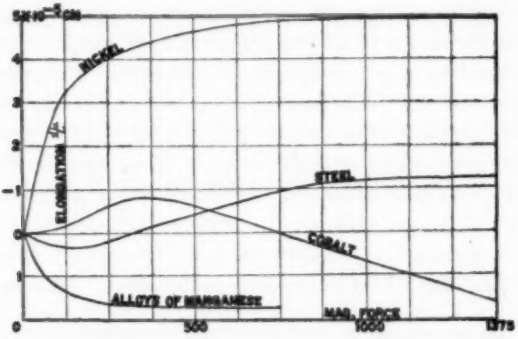

FIG. 13. *This graph illustrates the way in which various ferromagnetic substances change their length in a magnetic field. All values above the zero line represent contraction and all below indicate an elongation of the specimen.*

the Villari effect. Any of the methods for testing the magnetic properties of a substance are applicable for studying the Villari effect. For

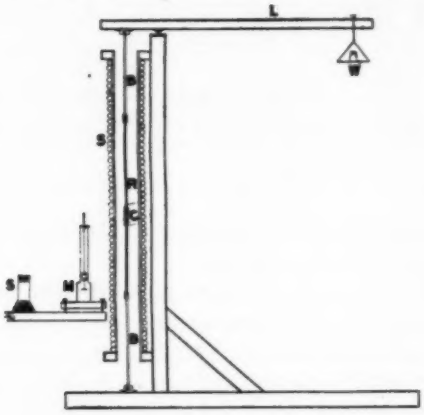

FIG. 14. *In the ballistic method the coil C is attached to a ballistic galvanometer and the inductive throw taken when the rod is stretched and again when unstretched. Without removing the rod the magnetization of the rod may be followed in the stretched and unstretched condition by means of the magnetometer M. S is the coil for compensating the field due to the solenoid S.*

absolute measures the ballistic methods are to be recommended, but for comparative work the magnetometric method commends itself. The

author has used (in yet unpublished work) a bell shaped magnet as a magnetometer. It swings in a massive cup of copper filled with fine transformer oil. The steadiness of the instrument makes it a useful means of studying wide ranges of intensities of magnetization when the rods are stretched and when normal. The more sensitive instruments, of the type recently described by Bozarth[54] are to be recommended where small variations in intensities need to be studied. As a rule it is best to get quantitative values with the ballistic method on the normal rod and then study relative values by means of the magnetometer when the rod is stretched and unstretched. Fig. 14 shows a combination of the two methods.

2. *Transverse Villari effect. Change in induction due to a normal stress.* The reciprocal of the transverse Joule magnetostrictive effect has had little or no study put upon it. This effect is the change in magnetic properties in a direction normal to the direction in which the elongation or compression is applied but parallal to magnetic field.

3. *Converse of the Guillemin effect. Change in induction due to bending.* To every magnetostriction effect there is a reciprocal effect. If this be true then there must be the converse of the Guillemin effect. This is another of those effects on which there has been little or no research work done. It is obvious that in this effect as in the Guillemin effect itself, there are a number of factors which it would be difficult to separate if a study was made of the effect.

B. CIRCULAR CHANGE

1. *Second Wiedemann effect. Longitudinal magnetization due to twisting a circularly magnetized rod.* Wiedemann[55] in investigating the second effect which bears his name, proceeded along lines similar to Wertheim;[55] a steel rod was placed in a torsion lathe, Fig. 15, with a magnetizing coil encircling the rod. In this effect the magnetizing coil instead of being attached to a battery is connected to a ballistic galvanometer. When a current flows along the rod it becomes magnetized circularly. Twisting the ferromagnetic rod thus magnetized, a longitudinal magnetization ensues which induces an electric current in the surrounding solenoid.

2. *The Wertheim effect. Circular magnetization due to twisting a longitudinally magnetized rod.* A steel rod is placed in a torsion lathe as in Fig. 15 and magnetized longitudinally by the surrounding solenoid. The terminals of a ballistic galvanometer are connected to the ends of the rod. When this rod is suddenly twisted an emf is developed at the

ends of the rod which gives rise to a current through the galvanometer. This means a current has been induced along the rod and that a circular magnetization was produced when the rod was twisted.

Wiedemann has studied quite extensively, not only the two effects found by him, but also the one discovered by Wertheim. Deservedly all three should be called Wiedemann Effects. These three effects should be restudied with standard specimens of ferromagnetic materials.

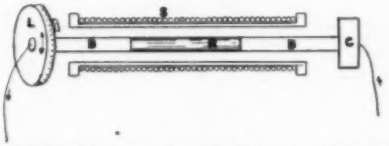

FIG. 15. *2nd Wiedemann Effect; When an electric current flows through the rod, R, it is magnetized circularly. Twisting the rod in this condition sets up a longitudinal magnetization which induces a current in the coil S.*

Wertheim Effect. When the rod, R, is twisted in a longitudinally magnetized condition a transient current will be set up along the rod while the twist is taking place. An ordinary magnet when twisted develops a potential difference between the two ends.

3. *Twisting rods with permanent torsional set and obtaining longitudinal and circular fields.* If magnetizing a rod with a permanent torsional set, both longitudinally and circularly, gives rise to a mechanical twisting of a rod, then we should expect that reciprocally a mechanical twisting of a permanently twisted rod would give both to a longitudinal and a circular magnetization. This is a field that needs investigation, and should be easily found if present.

C. VOLUME CHANGE

Nagaoka-Honda effect. Change in intensity of magnetization due to volume change. Here again the splendid work of Nagaoka[56] and his pupils have correlated these magnetic and mechanical characteristics of the above effect with those given under the Barrett effect. Just as the Villari effect is the reciprocal of the Joule effect so also the Nagaoka-Honda effect is a reciprocal relation of the Barrett effect. Figs. 16 and 17. In the first reciprocal relation it is the matter of length and in the second it is a question of volume. Like its reciprocal effect, the change in intensity of magnetization due to a change in volume is very small and extreme care must be exercised in its measurement. Very recently under the direction of Professor Bridgman, Mr. Yeh[66] has carried out some very careful experiments on this phase of magnetostriction in which much higher pressures were used than by Nagaoka and Honda.

ALLIED PHENOMENA.

There are a number of magnetic phenomena which are related to the magnetostrictive effects. Just how they are related is not clear, hence

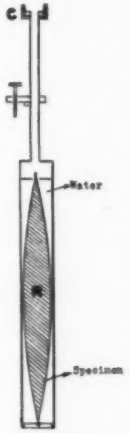

FIG. 16. *By means of the coupling C the cell containing the specimen R is connected to a compressor and the water forced in around R. This hydrostatic pressure changes the volume of R and in consequence the magnetic induction is also changed.*

there is a need for a study of the magnetostrictive effects and these allied phenomena on the same specimens of material in the hope that some of the relationships may be cleared up.

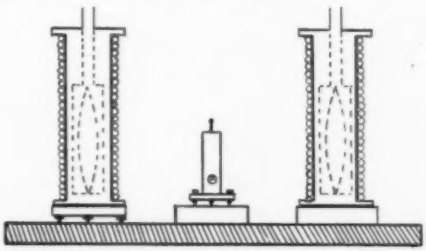

FIG. 17. *Two cells with specimens just as near alike as possible are arranged symmetrically on either side of a magnetometer. When the magnetometer has been balanced, pressure is put on one of the specimens and the sensitive magnetometer indicates the difference in magnetic induction due to pressure. The two cells and specimens are built as indicated in Fig. 16.*

1. *Change in resistance due to a magnetic field.* When a rod of any substance, (not necessarily ferromagnetic) is magnetized longitudinally or transversely it suffers a change in electrical resistance. The bismuth

spiral used in measuring magnetic field strengths is a good illustration. Bismuth is not a ferromagnetic substance hence the change in resistance in this case cannot be due to changes in dimensions as Wills[57] has shown that there is no magnetostrictive effect in bismuth. On the other hand Williams[58] and others have indicated in their work on steel that there is an analogy between the variation in resistance and the change in length due to a magnetic field.

The method that seems most adaptable to this study is the Kelvin Double Bridge.[59] Excellent outfits are now on the market.

2. *Change in thermo emf due to a magnetic field.* If a thermocouple is made up of copper and iron and the two junctions are maintained at two fixed temperatures a change in magnitude of the emf is developed by a magnetic field and this variation seems to bear some relation to the Joule effect for the iron wire. Lord Kelvin[60] showed that even a magnetized piece of iron showed a thermo-electromotive force when joined with an unmagnetized piece of iron. Kelvin's results indicated that the current flowed from weak longitudinally magnetized iron through the heated junction to the piece strongly magnetized in a longitudinal direction. Barus and Strouhal, Ewing, Bachmetjew and Grimaldi[61] have added other points of interest concerning this field. Analogously the Peltier and Thomson thermal-electric effects are also influenced by a magnetic field. Battelli, Houllevigue and Van Aubel[62] are some of the investigators in this field.

The thermoelectric effects thus produced by magnetization are in turn influenced by tension and other physical causes. Smith[63] has studied the effect of tension on thermoelectromotive forces by magnetization in nickel and in iron. The application of tension to nickel first lessens the thermoelectromotive force produced by a longitudinal magnetic field, but increases it for stronger fields. In the case of iron there is a parallelism between the effect of tension on the change of length and change of thermoelectromotive force due to a magnetic field.

3. *Emf's due to magnetization.* If two pieces of iron are placed in an electrolyte, a slight difference in emf between the two pieces of metal is observed due to the fact that it is impossible to find two specimens of metal which are exactly alike. If, however, one is magnetized while it is in the same electrolyte the emf will be considerably changed depending upon the conditions under which the experiment is carried out. In the case of iron, generally, the magnetized electrode is positive and the unmagnetized negative. Nickel is always positive for the magnetized electrode, while bismuth is just the reverse. Among those who have

contributed to this field are Gross, Andrews, Nichols and Franklin, Bucherer, Rowland and Bell, Hurmuzescu and Wyss.[64]

The maximum effect amounts to about 1/22 of a volt for iron and bismuth is only 1/20 as large. For the phenomenon that a magnetic field changes the emf of an ordinary cell it has been found that a fairly strong magnetic field produces a change of 1/20 to 1/5 of the original emf. Wyss[64] considers this change as a secondary effect due to a change in concentration of the electrolyte. This is a field that needs a great deal more study put upon it. One of the better types of potentiometers which are now upon the market makes a very good method for studying these effects.

4. *Change in the frequency of vibrating bodies when magnetized.* If a tuning fork or a steel rod clamped at the center is set in vibration the magnetization of either the fork or the steel rod changes the frequency of the vibration. It has already been discussed how magnetization changes the dimensions of a ferromagnetic substance and also in what ways the moduli of elasticity are modified. If these occur then changes in the frequency are to be expected. In the case of the tuning fork the frequency should also vary with the angle which the axis of the fork makes with the direction of the magnetic field and this is just what happens. Maurain[65] investigated the frequency of tuning forks in three different positions in a magnetic field. When the axis of the fork was normal to the field and the vibrations of the prongs were parallel, a field of 6350 gauss decreased the frequency 3.8 per cent. In the second case, where both the axis and the direction of vibration were normal to a field of 6530 gauss, the frequency was increased three-fourths per cent. When the fork was finally placed with the axis parallel to the field, then no matter how it was oriented the direction of vibration was normal to the field and for a field of 1090 gauss the frequency was increased 0.38 per cent. In a similar fashion the longitudinal vibrations of a steel rod should show a difference in frequency between the magnetized and unmagnetized condition. It would be possible to show this by some Lissajou figure arrangement.

5. *Magnetic tick.* When a rod is suddenly magnetized a distinct click is heard. Honda and Shimizu[66] in discussing the sound produced in a steel rod when magnetized put them in three classes. The first is the combined effect of the magnetic force and the electric current. Page, Marrian and De La Rive[67] observed this kind of noise. The kind of noise particularly referred to here is that observed by Maurain, Wertheim and Matteucci[68] which they ascribe to the sudden expansion or

contraction of the rod being magnetized. A third noise is that in which a sudden impulse thus given to the rod sets it into vibration and causes it to give forth a distinct note, but which very quickly dies down. It appears rather evident that magnetostrictive effects play certain rôles in these phenomena and which need to be restudied in a more quantitative fashion than has yet been done.

6. *The Barkhausen effect.* Barkhausen[69] connected a coil having a large number of turns to a radio amplifier. When a thin strip of iron was placed in the coil and a magnetic field was applied, a murmuring could be heard in the telephone receiver, as though during the process of magnetization it had not gone forward continuously but rather by jumps. There are indications[70] that this bears some relation to the magnetostrictive effects. For instance, in the case of the Joule magnetostrictive effects in silicon-steel crystals[71] there are sudden changes in the length of the crystals. Wherever these occur, there the Barkhausen noise swells out the loudest. This point is being studied more carefully in hopes that some definite correlation may be obtained.

There are doubtless many other effects which could and should be correlated with the phenomena of magnetostriction, but these will at least indicate the direction in which to go.

Magnetostriction must be included as one of the phenomena for which any comprehensive atomic theory must find a reason. The simple mechanical effects which occur as for instance in the Joule effect ought somehow to throw light on atomic structure. What is needed in the whole field of magnetic phenomena is a discovery that will open up the field of magnetism as the discovery of the x-rays did the electromagnetic field.

This résumé of previous work in the field of magnetostriction is by no means an exhaustive one, but it is hoped that sufficient leads have been given to enable any one to pursue the investigation in a thorough going fashion.

Just as this paper goes to press, the excellent review of magnetostriction by McKeehan[72] appears and will be found of very great interest.

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Magnetization of single crystals of iron. The — iron used in the experiments was exceptionally pure iron, manganese to the amount of 0.4 per cent being the most abundant impurity; the carbon, originally present in amount 0.1 per cent, was eliminated by long-continued heating in moist hydrogen. Slabs of the material 26 by 7 by 0.6 cm were extended in proportions varying from 2 per cent to 10 per cent, and annealed for three or four days at 880°; after this process the size of the crystals depended on the amount of the initial extension, and the entire slab might become a single crystal if the extension had been 2.75 or 3 per cent. A rod cut from such a slab, having its longitudinal axis inclined at 21° 10′ and 70° 10′ to the tetragonal axes of the crystal, displayed a magnetization-curve consisting of three straight segments connected by sharp bends; on attaining the third segment the iron was saturated with a magnetization of 1707, agreeing extraordinarily well with the value for polycrystalline iron given by Weiss. The hysteresis-loss was extraordinarily small, the initial permeability extraordinarily great. Measurements were made upon ellipsoids cut from slabs of iron having the 100 plane, or the 110 plane or the 111 plane, in the plane of the slab; these ellipsoids being very thin (oblate) the allowance for demagnetizing-factor had to be enormous. In brief, the results of these measurements are as follows: the magnetization is parallel to the magnetizing field if this is directed along the tetragonal, trigonal, or digonal axis; otherwise there is a component of magnetization perpendicular to the magnetizing field, which however is always much smaller than the parallel component, so that the magnetization deviates from the field by an obliquity never exceeding 11° and usually much less. The deviation is very slight if the magnetizing field amounts to but a few gauss or if saturation is nearly attained; it passes through a maximum at intermediate field strengths. Curves of the parallel and perpendicular components of magnetization as function of orientation in the 100, 110, and 111 planes, for each of several values of fieldstrength, are given in the article; they are periodic, with period of 180° in the 110 plane, 90° in the 100 plane, and 60° in the 111 plane. [K. Honda and S. Kaya (Tokyo); Tohoku Univ. Sci. Rep., *15*, pp. 721–753; 1926].

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A CALIBRATED THERMAL RESISTANCE PLATE FOR MEASURING HEAT FLOW

By Carl G. F. Zobel

ABSTRACT

A brief review is given of the various methods considered for measuring heat flow. The heat flow meter as developed for heat transfer measurements is described and data presented substantiating the basic claims made for it. The calibrating apparatus, the constancy of calibration, and the application of the meters to actual measurements of heat flow through a roof are included in the paper.

INTRODUCTION

Several years ago, the Laboratory undertook the development of an instrument or method for the measurement of heat flow. This was necessitated by the interest and research in heat transfer.

The methods considered were the hot-air box, the guarded vapor box, and the electrically heated plate.

In the hot-air box method, a box is made of the material to be tested. Heat is supplied to the interior of the box usually by electrical means such as, a resistance coil or lamp bank. It is customary to use a fan to circulate the air in order to maintain a uniform temperature throughout the interior of the box. The electrical input to the heater and the fan motor is a measure of the heat transmitted through the material.

The guarded vapor box is similar to the hot-air box. In this method, a box is so constructed and thermally balanced, that the supplied heat only flows through the specimen which forms one side of the box.

Because of the complicated structure of the hot-air box and the guarded vapor box, they were discarded in favor of the electrically heated plate. In this latter method, a guarded flat electrical heating element, clamped between two metal sheets, was the source of heat. Slabs of the material to be tested were placed in intimate contact on both sides of the heater. The heat passed through the slabs into water-cooled plates which were placed against the outside surfaces of the slabs. Since the dimensions of the plate were known, the conductivity could be determined from the temperature drop through the material, and from the electrical input.

However, this method was not adaptable for field use, that is, for thermal measurements on walls or other parts of a building already erected.

The idea of using a calibrated thermal resistance plate was introduced. This was the inception of what was to become the heat flow meter.

Although this means of thermal measurement was not unknown to investigators in other laboratories, the first consistent and successful application of this method occurred at this Laboratory.

Particular attention will be given to the description of the meter and its calibration, and to the present work on the constancy of calibration and application of the heat flow meter. In addition some comments on the effect of the reflecting power of surfaces on heat flow through roofs will be included.

Theory of Heat Flow Meter. If the thickness and thermal resistance of a thin slab of material be known, the heat flow into a surface can be measured by placing this material against the surface and determining the temperature gradient through it. This would constitute a heat flow meter. If it were calibrated, quantitative results could be obtained, for the heat flowing through the surface flows through the meter and from the relation

$$H = KA\frac{d\theta}{dx} \text{ where}$$

H = time rate of heat flow
A = area
x = distance measured normal to the surfaces of the slab.
K = thermal conductivity of the material
$\frac{d\theta}{dx}$ = the temperature gradient

the heat flow could be determined. The above equation is for a flat homogeneous slab where the temperatures are maintained constant and uniform over the surfaces. The conductivity K varies slightly with temperature; however if K for the temperature concerned is used:

$$H = KA\frac{\Delta\theta}{d}.$$

where $\Delta\theta$ = the temperature difference between opposite faces of the slab.
d = the distance between opposite faces of the slab.

In the case of the heat flow meter it was more exact and convenient to use the form:

$$H = CA\Delta\theta$$

where C = the conductance.

Description of the Heat Flow Meter.[1] The heat flow meter was constructed as follows: The thermal resistance plate was a thin sheet of

[1] Transactions A. S. H. V. E., *30*, pp. 65–104; 1924.

formica $\frac{1}{16}''$ to $\frac{1}{8}''$ thick and two feet square. The effective area was $15\frac{1}{4}'' \times 15\frac{1}{4}''$, the remaining $4\frac{3}{8}''$ strip being used as a guard ring.

The temperature drop through the effective area was measured by differential thermocouples which were connected in series and uniformly distributed. As many as two hundred couples were used in some of the meters.

The surface or reference temperature was determined from a series of thermocouples cemented to the surface of the plate in such a manner that the readings gave the average surface temperature.

Calibration of heat flow meter. Method. The method of calibrating the meters was to pass known amounts of heat through them and observe the electromotive force registered by the two thermocouple systems. The readings of the differential thermocouples varied linearly with the surface thermocouple readings.

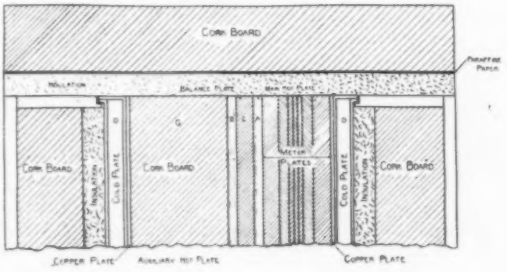

FIG. 1. *Calibration apparatus.—Side view.*

In order to accomplish this calibration, the set up used was the usual thermal conductivity apparatus modified so that the measured heat flow was presumably unidirectional.

Calibration apparatus. Fig. 1 is a sketch of the apparatus where unidirectional heat flow was secured by means of a balanced hot plate. The diagram is self-explanatory. When the temperature potential between the guarded main heater *A* and the guarded auxiliary heater *B* was zero, there was no heat flow from the one to the other. Hence if the heat supplied to *A* was constant and sufficient heat was supplied to *B*, so that the temperature potential between the two was zero, all the heat from *A* must flow through the heat flow meters.

The electrical control is indicated in Fig. 2. A slide wire resistance was connected in series with each tube rheostat for fine adjustments.

The ammeter and voltmeter were used for approximate control only, the ultimate values of current and voltage being determined by a potentiometer system.

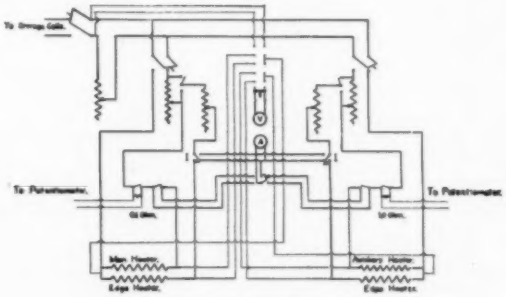

FIG. 2. *Electrical control.*

The water for the cold plates was pumped from a tank to the main cold plate. From there it flowed to the auxiliary cold plate then back to

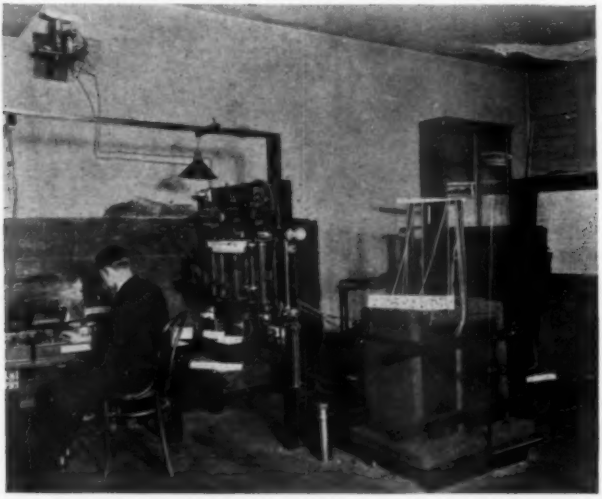

FIG. 3. *Calibration apparatus.*

the supply tank. The water in this tank was constantly stirred and kept at a given temperature by regulation of a heating element and ice water inlet.

Calibration data. The calibration involved the relation of temperature gradient to heat flow and the change of this relation with actual temperature. The difference in temperature of the two faces of the meter was measured in terms of differential microvolts and the actual temperature in surface microvolts.

Accuracy. The heat flow meter has proved to be very sensitive and accurate to within plus or minus two per cent of the heat flow.

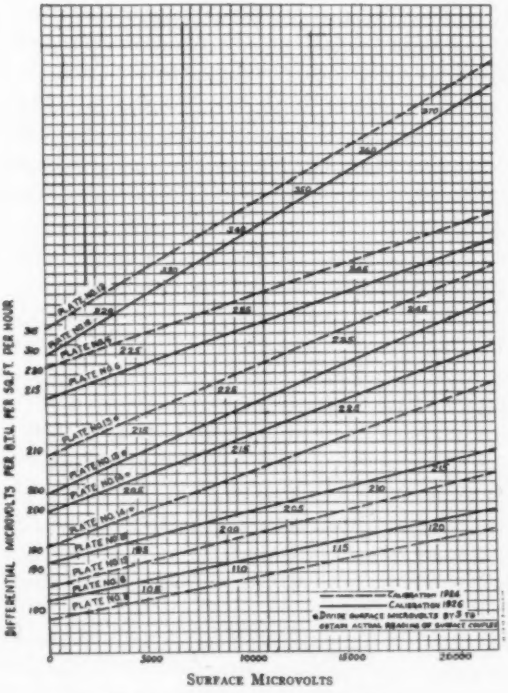

FIG. 4. *Calibrations for meter plates.*

Constancy of calibration of heat flow meters. After a lapse of two years, the meters were recalibrated to check the constancy of their calibration. Calibration runs were made at three different temperatures and the resulting calibrations were compared with those obtained two years ago.

It will be noted that the two calibration lines for the same meter did not coincide. However, upon further examination, the more recent

calibration was found not to vary over five per cent and more often two per cent from the previous one. This was accounted for in the set up, because a larger air space was enclosed between the plates in the last calibration. The air space was used in an attempt to secure uniform distribution of the heat flow normal to the surface of the meters. If the meters were clamped tightly against each other, there would be points of contact and therefore more heat flow at those points. If a thermal resisting medium were placed between the meters, the effect of these contact points due to unevenness of the surface would be reduced. Such was the indicated result in previous calibration tests.

The purpose of the recalibration was primarily to discover any change in the meters themselves due to aging of the material or any unpredicted cause. From the calibration lines (Fig. 4), it was apparent that the plates retained their thermal properties for a considerable length of time. A commercial use of these plates may therefore be possible in so far as constancy of calibration is concerned.

The calibration of the meters having been checked, an application of them seemed desirable.

Application. If a meter were fastened to a surface, the heat flow through that surface could be determined. In the case of a roof the heat flow from the roof into the attic could be measured by fastening a heat meter to the under surface or attic side of the roof. However, the thermal resistance of the plate itself would affect the result. This effect can be eliminated by applying a relation determined from the temperatures on both sides of the roof over the meter, and at a small distance away from the meter. Suppose H_1 were the heat flow through the meter and $\Delta\theta_1$ the temperature drop through the roof above the meter. The heat flow H_2 through the roof without the meter would be

$$H_2 = H_1 \frac{\Delta\theta_2}{\Delta\theta_1}$$

where $\Delta\theta_2$ was the temperature drop through the roof at a distance away from the meter.

The air temperatures and surface temperatures were determined at the same periods. From these data it was possible to obtain: 1. Temperature gradient—Temperature difference per unit distance assuming the material to be made of homogeneous sheets. 2. Conductivity—Time rate of heat flow per unit area per unit temperature gradient. 3. Conductance—Time rate of heat flow per unit area per unit difference

in temperature between opposite faces of a slab when the direction of heat flow is normal to the faces of the slab. 4. Thermal resistance—The reciprocal of conductance. 5. Transmittance—Rate of heat flow per unit area per unit temperature difference between the medium on one side of a wall and the medium on the other. The temperatures are measured far enough from the material so that the effect of the latter can be neglected. 6. Surface conductance—Rate of heat transfer between a surface and the surrounding air, per unit area and unit temperature difference between the surface and the air.

Heat Flow through a slate roof. Since data were apparently lacking in regard to thermal conductance of roofs, four heat meters were installed in the attic of the U.S. Bureau of Mines. Three were fastened to the under or attic side of the slate roof, one on each of three different exposures, east, south, and north. The fourth was clamped to the top or attic side of the ceiling of a room underneath. Multiple thermocouples were cemented to the roof surface and to the attic surface of the roof immediately above the plate and surrounding the plate at a distance of about eight inches. Air temperatures were measured five inches from each surface.

ATTIC INSTALLATION.

The roof is composed of $2\frac{1}{4}''$ tongue and grooved boards covered with slater's felt and $\frac{3}{16}''$ thick slate.

The plaster ceiling was keyed to heavy wire netting, which is supported by $\frac{3}{4}''$ square iron rods set at 12″ centers. The plaster is about $\frac{3}{4}''$ thick and has a rough, lumpy, upper side which is covered with a thick layer of dust.

Fig. 5 shows one meter mounted against the attic surface of roof and location of thermocouples. The meter for measuring heat flow through the ceiling was mounted in the same manner and is also shown in Fig. 5.

All thermocouple leads were well insulated and conducted to the potentiometer outfit where they were soldered to a multiple point thermocouple switch. This enabled the observer to easily and quickly obtain the particular reading desired. The emf's were determined by a Tinsley potentiometer having a microvolt dial.

It was immediately realized that due to the natural variations of the inside and outside temperatures, it would be necessary to make observations over a time period of such length that the change in heat content of the material would become a negligible factor or that sufficient data be obtained to correct for the change in the heat content of the material.

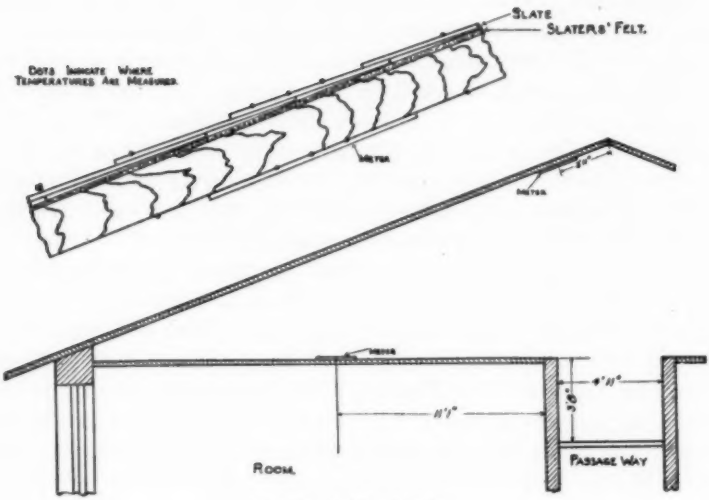

FIG. 5. *Attic installation.*

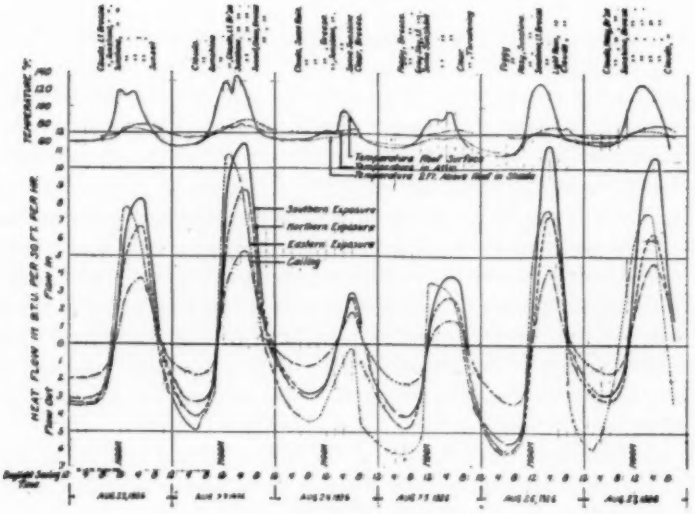

FIG. 6. *Heat flow curves for slate roof.*

Observations were taken for a period of ten days. During that period the days were warm, the nights cool, and the other conditions changed about from sunshine to rain to sunshine with and without wind. A log of the last six days of this period is shown in Fig. 6.

Results and discussion. The curves in the log of Fig. 6 for the heat flows as actually measured by the heat flow meters showed that the peaks or maximum flows for the different exposures occurred in the time order of east, north, and south. According to the peaks, the order of the amount of heat flow through the roof was south, east, then north. These results could be expected since in the morning the sun's rays strike the east roof more nearly at right angles than during any other part of the day. The incident angle of the rays upon the north roof was always greater than that of the other two exposures and the time for the sun effect was less. Thus the data secured corroborated the predictions.

The values of the different heat transfer factors for the roof obtained from this work and the corresponding values for previous determinations are given below in Table 1.

TABLE 1.

	Roof		Ceiling	
	(1)	(2)	(1)	(2)
ϕ_m	44°F	94°F	72°F	75°F
C	0.35	0.36-0.37		1.64
U			0.60	0.57
h_1				2.07
h_2	1.3	1.25	1.4	1.47

ϕ_m = Average temperature of material °F

C = Conductance B. t. u. per sq. ft. per hour per °F between faces of material

U = Transmittance B. t. u. per sq. ft. per hr. per °F between air temperatures.

h_1 and h_2 = surface conductances B. t. u. per sq. ft. per hr. per °F between surface and air temperatures.

[1] Values obtained in March

[2] Values obtained in August

The values which were obtained with the heat flow meter under the severe conditions stated above agree with those obtained before under uniform conditions. This agreement was another indication that the heat flow meter, its application, and the computation of results constituted a reliable method for determining heat flow and thermal properties of materials.

The weather varied to such an extent during the test that the heat capacity of the roof was equivalent to 50 per cent of the heat flow for

one day. The meter measured only the flow from the attic surface of the roof to the attic air. The total heat flow therefore was determined by adding a correction which involved the change in heat capacity of the roof during the cycle considered. This correction may be considered as $f(R, \Delta\theta_m, t, \Delta\phi_m)$

If H_m = average heat flow

t = time

$\Delta\theta_m$ = average temperature drop from surface to surface

$\Delta\phi_m$ = difference in average roof temperature for period t.

then

$$C=\frac{H_m\times t+R\times\Delta\phi_m}{\Delta\theta_m\times t}=C_1+\frac{R\times\Delta\phi_m}{\Delta\theta_m\times t}$$

where C_1 was the partial conductance determined by the heat meter on the under or cold side of the material.

$R=\delta ws$

δ = condition factor

w = weight of material in lbs. per square foot.

s = specific heat

This correction was determined by observations of heat flow for two cyclic periods when there was no great variation in weather conditions. With same approximate cyclic temperatures and weather conditions R and C may be determined from the two equations. If for the same material the two equations were:

$$C=\frac{H_{1m}\times t_1+R\times\Delta\phi_{1m}}{\Delta\theta_{1m}\times t_1}=C_1+\frac{R\times\Delta\phi_{1m}}{\Delta\theta_{1m}\times t_1}$$

$$=C_1+\Delta C$$

$$C=\frac{H_{2m}\times t_2+R\times\Delta\phi_{2m}}{\theta_{2m}\times t_2}=C_2+\frac{R\times\Delta\phi_{2m}}{\Delta\theta_{2m}\times t_2}$$

$$C_1-C_2=\frac{R\times\Delta\phi_{2m}}{\Delta\theta_{2m}\times t_2}-\frac{R\times\Delta\phi_{1m}}{\Delta\theta_{1m}\times t_1}$$

$$C_1-C_2=R\left(\frac{\Delta\phi_{2m}}{\Delta\theta_{2m}\times t_2}-\frac{\Delta\phi_{1m}}{\Delta\theta_{1m}\times t_1}\right)$$

$$R=\frac{C_1-C_2}{\dfrac{\Delta\phi_{2m}}{\Delta\theta_{2m}\times t_2}-\dfrac{\Delta\phi_{1m}}{\Delta\theta_{1m}\times t_1}}$$

If the correction (R) were determined from data for two days observations and then applied to the computation of C from data for other days during the same period of test, then there will be some check on the reliability of this mode of correction. This was best illustrated by the values of heat conductance for a slate roof.

TABLE 2

DATE	C_1	ΔC	C
Aug. 22, 1926	0.22	0.13	0.35
Aug. 23, 1926	.24	.13	.37
*Aug. 24, 1926	.23	.23	.46
Aug. 25, 1926	.21	.16	.37
Aug. 26, 1926	.20	.16	.36
Aug. 27, 1926	.24	.12	.36

* Observations for August 24, 1926 should be completely disregarded as from the very nature of the conditions of test on that day it was obviously futile to obtain any data of merit.

With the exception of August 24, 1926, the resulting values of C were very consistent. Since the data for that day were disregarded before computations were made, it need not be considered. Apparently this method of correction, although approximate was sufficiently accurate for this work.

Due to the heat energy of the sun the outside surface of the roof was at a higher temperature than either the outside or the attic air. Consequently there would be an inflow during the day and outflow or heat loss at night. Under these conditions it was inadvisable to attempt to measure the heat transmission.

However the heat transmission through the ceiling of the room underneath the attic could be measured. If the customary equations were applied:

$$U = \frac{1}{\sum_1^n \frac{d_n}{K_n} + \frac{1}{h_1} + \frac{1}{h_2}}$$

where U is the transmittance, and h_1 and h_2 the respective surface conductances on the two sides, d_n and K_n are the respective thicknesses and conductivities of the different layers. The heat flowing per unit time through the area A in equilibrium with the temperature difference $\Delta\theta$ is equal to

$$H = \frac{A\Delta\theta}{\frac{1}{C} + \frac{1}{h_1} + \frac{1}{h_2}}$$

where C is the conductance, then equating for U

$$U=\frac{H}{A\Delta\theta}=\frac{1}{\frac{1}{C}+\frac{1}{h_1}+\frac{1}{h_2}}$$

The equation in the form furnished an easy and convenient check on the values for conductance, surface conductance, and transmittance, as independently determined in these tests.

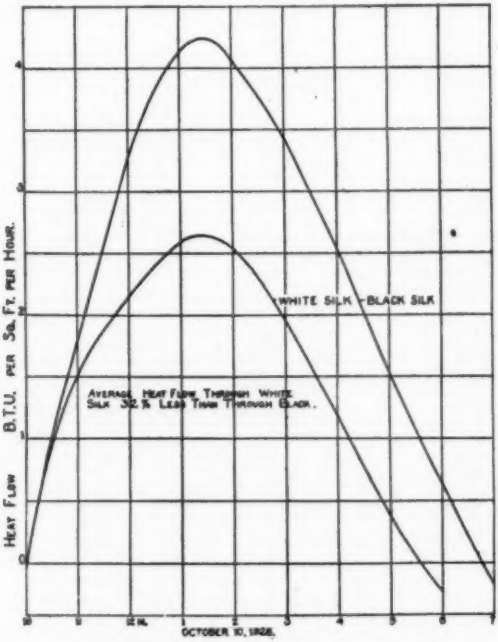
HEAT FLOW B.T.U. PER SQ. FT. PER HOUR.
WHITE SILK
BLACK SILK
AVERAGE HEAT FLOW THROUGH WHITE SILK 32.5% LESS THAN THROUGH BLACK.
OCTOBER 10, 1928.

FIG. 7. *Effect of white and black silk on heat flow.*

The only example available for this check was in the case of the ceiling where the quantities measured were:

$$C=1.64 \quad h_1=2.07 \quad h_2=1.47 \quad \frac{H}{A\Delta\theta}=0.60$$

$$U=\frac{1}{\frac{1}{1.64}+\frac{1}{2.07}+\frac{1}{1.47}}=.57$$

using the above equation. But according to measurement $U=0.60$, therefore,

$$U_m = .585 = .59$$

or a deviation of plus or minus 2.5 per cent from the mean.

Effect of reflecting surfaces on heat Flow. Heat flow meters were employed to determine the magnitude of the effect of different reflecting surfaces upon heat flow. Meters were mounted on the attic side of a

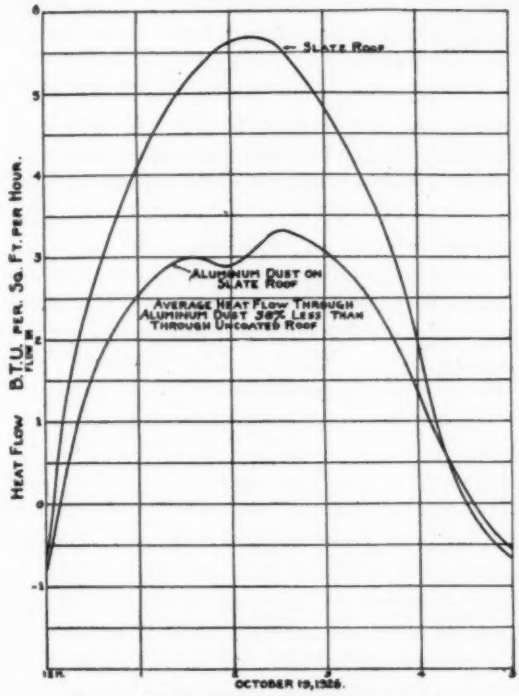

FIG. 8. *Effect of aluminum powder on heat flow.*

slate roof which had a southern exposure. The customary thermocouple system was installed.

Black silk pongee was placed on the roof above one heat meter and white silk pongee was placed on the roof above the other. Observations were taken for one day. The data were plotted and the per cent difference in heat flow between the two portions of the roof noted. The

same effect was determined for aluminum powder. The heat flow curves are given in Figs. 7 and 8.

The white silk reduces the heat flow 32 per cent and the aluminum powder 38 per cent. In the case of the aluminum powder, it was merely brushed on the slate without using bronzing liquid or a binder of any sort.

Conclusions. The results of the investigation reported in this paper may be summed up briefly as follows:

Heat flow meters as constructed are reliable and sufficiently constant in their calibration to possibly warrant their use in commercial laboratories.

Applications of the heat flow meters give consistent results. No important objections are apparent so far. Conductance values determined for a slate roof and ceiling check former values.

It is necessary to correct for cyclic changes in the heat capacity of the material under test in order to secure the total heat flow since the plate only measured partial or surface flow.

The use of a reflecting roof surface reduces the heat flow to a considerable extent. This may be used to advantage in the summer.

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VELOCITY SELECTOR FOR ATOMIC RAYS

By J. Tykocinski-Tykociner

ABSTRACT

The description of an apparatus for selecting atoms of particular velocities from atomic and molecular rays is given. The theory of the action of the apparatus is outlined and the characteristics of expected spectrum-like images are discussed. An atomic ray on its way towards a chemically active target is conveyed through two vibrating slits at a distance D apart. The vibrations are controlled by the oscillating current of an electron tube. The emerging ray contains selected velocities. For very small ratios of the width w of vibration to the amplitude of the slit a, the selected velocities are determined by $v_n = 2Df/n$ where f is the frequency of vibration and $n = 0, 1, 2, 3$, etc. For larger w/a ratio a great number of velocity bands are obtained each embracing a range

$$v_n' - v_n'' = 2Df\left[\frac{1}{n - w/2\pi a} - \frac{1}{n + w/2\pi a}\right]$$

When used in connection with the Gerlach and Stern magnetic moment analyzer, images should be obtained consisting of n pairs of lines, whose thickness is

$$d_n = \frac{1}{4}\frac{M}{m}\left(\frac{\delta H}{\delta S}\right)_0 \frac{l^2}{D^2}\frac{n}{f^2}\frac{w}{\pi a}\left(K + \frac{1}{2}\right)$$

An example for a hydrogen atom ray at 500° K is calculated.

For the study of the properties of streams of molecules or atoms it is often required to know the law of distribution of velocities within the stream and to select out of a variety of velocities a particular one. In a recent paper[1] John B. Taylor, discussing the sources of errors in connection with the measurement of magnetic moments of atoms, emphasizes the desirability of producing a ray of single or narrow range velocity atoms. Due to the encouragement of Prof. Jakob Kunz a velocity selector is being developed for this purpose. The following is a description of the principles underlying a method of selecting atoms of any desired range of velocity and of detaining all the rest.

Let a stream of neutral atoms (Fig. 1) emerging from the source S_0 enter an evacuated space through the slit S_1. The stream reaching the slit S_2 contains atoms of a large variety of velocities. The holder of the slit S_2 is suspended between two phosphor bronze strips R perpendicular to the magnetic field H of an electromagnet. These strips are inserted in the circuit of an electron tube oscillator V whose frequency (2000–20000 per sec.) is controlled by means of a variable condenser C and inductance coil L. Electrodynamic forces set the strips R with the slit S_2 into vibration at a frequency determined by the constants of the circuit. The holder of the slit S_2 crossing periodi-

[1] Magnetic Moments of the Alkali Metal Atoms, Phys. Rev., *28*, p. 582; 1926.

cally the path of the atomic ray acts as a shutter breaking the continuous stream of atoms into short and narrow intermittent portions. The fractions of a half period during which atoms can enter through S_2 is

$$t_1 = wT/2a\pi$$

where w is the width of the slit, a the amplitude and T the period of vibration. At a distance D from S_2 another system with a slit S_3 of similar construction and inserted in the same oscillating circuit is vibrating with the same frequency, in opposite direction. The time

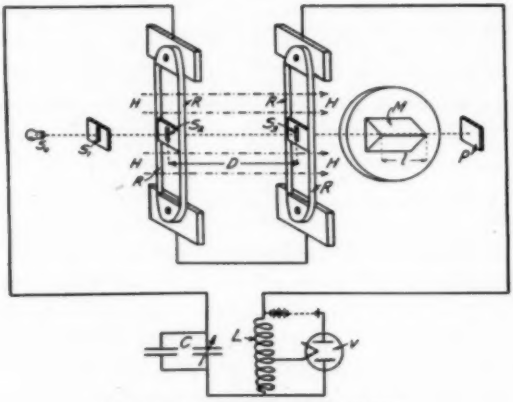

FIG. 1. *Diagram of the velocity selector.*

fraction of a half period during which the slit S_3 remains open for a stream moving along the line S_0–S_3 is

$$t_2 = t_1 = w/a2\pi f \tag{1}$$

Of all the atoms which pass the vibrating slit S_2 only those can enter through the slit S_3 which arrive there at moments when the slit S_3 crosses the line S_0–S_3, i.e., during the time t_2. During that time groups of atoms arrive having within a narrow band the following selected velocities

$$v_1 = \frac{D}{T\,1/2} \quad v_2 = \frac{D}{T\,2/2} \quad v_3 = \frac{D}{T\,3/2} \quad v_4 = \frac{D}{T\,4/2} \cdots v_n = \frac{D}{T\,n/2} \tag{2}$$

The term "selected" is given to that velocity with which atoms can pass through the selector, vibrating with a chosen frequency f and with an amplitude a, imagined to be so large in comparison with the

width w of the slit, that the time t_2 is very small. The relative values of the selected velocities are:

$$v_1:v_2:v_3:v_4:\cdots v_n = 1:1/2:1/3:1/4:\cdots 1/n \tag{3}$$

For all practical cases t_2 will be from $T/50$ to $T/5$. Consequently the selected velocities will extend into ranges of higher and lower velocities. Each of the ranges extends over a region of velocities, whose upper and lower velocity limits v_n' and v_n'' are respectively

$$\begin{aligned} v_n' &= \frac{D}{T\,n/2 - t_2/2} = \frac{D}{T\,n/2 - wT/4\pi a} = \frac{2Df}{n - w/2\pi a} \\ v_n'' &= \frac{D}{T\,n/2 + t_2/2} = \frac{D}{T\,n/2 + wT/4\pi a} = \frac{2Df}{n + w/2\pi a} \end{aligned} \tag{4}$$

and either of the relations

$$\begin{aligned} \frac{v_n'}{v_n''} &= \frac{n + w/2\pi a}{n - w/2\pi a} \\ \frac{v_n'}{v_n} &= \frac{1}{1 - w/2\pi na} \\ \frac{v_n''}{v_n} &= \frac{1}{1 + w/2\pi na} \end{aligned} \tag{5}$$

can be used for the calculation of the velocity range embraced by each band.

Equations (2), (3) and (4) are valid also for $n=0$, in which case the velocity band extends over a range from $v_0' = \infty$ to $v_0'' = 2D/t_2 = 4\pi fDa/w$.

Thus it follows that the stream of atoms emerging from the slit S_2 does not contain atoms of all velocities. Moreover certain wide ranges of velocities are automatically cut out by timing the shutter-like action of the vibrating screen holding the slit S_3. Leaving the distance D unchanged, the selected velocities can be chosen by adjusting the frequency of the oscillating current driving the vibrating shutters, while the width of each velocity band can be controlled by conditions which determine the amplitude of vibrations, i.e., the intensity of the oscillating current and the current exciting the magnetic field H.

In Fig. 2 the relations between the selected velocities v_n filtered out by the selector and the periodic motion of the slit S_3 are shown graphically. The abscissas represent the time in units of half periods $T/2$ of

the vibrator. The ordinates represent the distances in units D traveled by the atoms having particular velocities:

$$v_0 = \tan \alpha_0 = \infty \; ; \; v_1 = \tan \alpha_1 = \frac{D}{0.5\,T} \; ; \; v_2 = \tan \alpha_2 = \frac{D}{T} \; ; \; v_3 = \tan \alpha_3 = \frac{D}{1.5\,T}$$

The tangents of angles $\beta_1, \beta_2, \beta_3$, etc., and $\gamma_0, \gamma_1, \gamma_2, \gamma_3$, etc., determine, respectively, the upper and the lower velocity limits for each selected

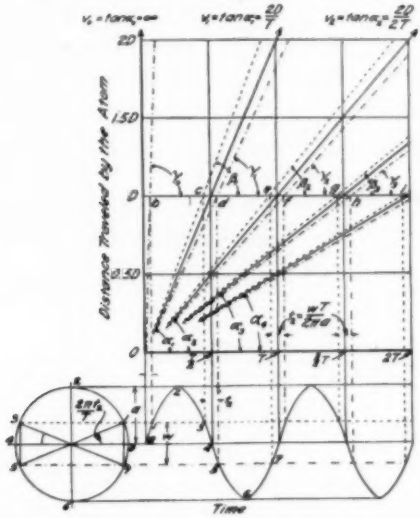

FIG. 2. *Characteristic of the velocity selector.*

velocity of the atom stream leaving the slit S_3. The time intervals c–d, d–f, g–h, i–j, etc., are all equal to $t_2 = wT/a2\pi$ and correspond to that fraction of a period within which atoms having selected velocity bands v_0, v_1, v_2, v_3, etc., arrive at the slit S_3 after having traveled the distance D. From the graph it is evident that $\sin(\pi t_2)/T = w/2a$. From the latter the relation (1) is obtained considering that the ratio t_2/T is small.

Let us consider for instance a stream of atomic hydrogen in an evacuated vessel, out of which it is required to select atoms of a predominating velocity $v_1 = 3600$ m/sec. The source is then advantageously sustained at a temperature of 500°K to supply atoms of the most probable velocity equal to 3600 m/sec. For $D = 20$ cm the slits S_2 and S_3 must vibrate with a frequency $f = v_1/2D = 3600/2 \times 0.2 = 9000$ per second.

The only groups of atoms which can pass the selector will belong to narrow velocity bands having the following selected velocities:

$v_1 = 3600\ m/\text{sec.}$ $v_2 = 1800\ m/\text{sec.}$ $v_3 = 1200\ m/\text{sec.}$ $v_4 = 900\ m/\text{sec.}$

Assuming the slit S_3 to have a width of $w = 0.01$ cm and the intensity of the oscillating current and of the magnetic field so adjusted that the amplitude of vibration of the slits is 0.05 cm, the limiting velocities v_n' and v_n'' for each band were calculated from the relation (4). The results are included in the table shown below.

The values N in the sixth column represent relative numbers of selected atoms carried within each band, as compared with a total number of 10000 atoms transmitted when the slits S_2, S_3 were at rest. Maxwell's law of distribution of velocities was assumed to hold in this case.

If the atoms emerging from the slit S_3 (Fig. 1) possess magnetic moments, their velocities can be analyzed by splitting the atomic ray in the field of an electromagnet M. Like in the classical experiments of Stern and Gerlach, a characteristic image of chemical action or condensation is produced by the deflected rays striking a sensitized

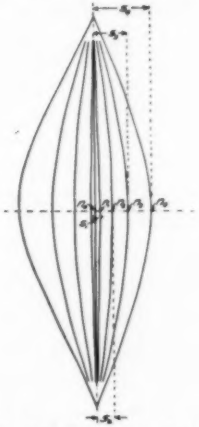

FIG. 3. *Diagram of image indicating distribution of atomic rays of selected velocities.*

target P. However, the usual images obtained with streams containing atoms of all velocities show two branches, their densities shaded gradually from maximum to minimum. The longer the exposure the denser become the branches and the wider extend their shadings. The magnitude of the deflection thus produced is not clearly defined.

The purpose of the velocity selector is to change this image into a more definite spectrum-like picture. Instead of one pair of hazy branches, the picture will present the following characteristics, illustrated in Fig. 3.

a. A number of distinct branches, n_0, n_1, n_2, etc., will appear, separated by white spaces, each branch corresponding to a narrow range of velocities, determined from (4)

$$v_n' - v_n'' = 2Df\left(\frac{1}{n - w/2\pi a} - \frac{1}{n + w/2\pi a}\right) \tag{6}$$

b. The deflections of the bands following each after another will increase outward with the square of n. The magnitude of the deflections $s_1, s_2, s_3 \cdots s_n$ can be calculated as follows.

For the magnetic moment M of an atom measured by the deflection of an atomic ray in an inhomogeneous magnetic field, W. Gerlach and O. Stern,[2] gave the relation.

$$\frac{1}{M} = \frac{1}{2}\,\frac{1}{S}\left(\frac{\delta H}{\delta S}\right)_0 \frac{l^2}{3.5RT}\left[1 + \frac{1}{12}\,\frac{\left(\frac{\delta H}{\delta S}\right)_l - \left(\frac{\delta H}{\delta S}\right)_0}{S}\,\frac{Ml^2}{3.5RT}\right]$$

Solving this equation for S and substituting for the rather uncertain term $3.5\,RT$, the quantity mv_n^2 where m is the atomic weight and v_n any one of the selected velocities, of the atoms, we obtain for the n-th branch the deflection

$$S_n = \frac{1}{2}\,\frac{M}{m}\left(\frac{\delta H}{\delta S}\right)_0 \frac{l^2}{v_n^2}\left\{\frac{1}{2} + \sqrt{\frac{1}{12} + \left(\frac{\delta H}{\delta S}\right)_l \Big/ 6\left(\frac{\delta H}{\delta S}\right)_0}\right\} \tag{7}$$

The velocity v_n is predetermined by equation (2). All that is necessary to know is the distance D and the frequency $f = 1/T$. Both can be measured with great precision and f can be kept constant for any length of time by means of piezoelectric oscillators. Substituting for v_n in equation (7) its value from (2) and denoting as K the expression under the radical sign which is always smaller than $1/2$, the relation is obtained

$$S_n = \frac{1}{8}\,\frac{M}{m}\left(\frac{\delta H}{\delta S}\right)_0 \frac{l^2}{D^2}\,\frac{n^2}{f^2}\left(K + \frac{1}{2}\right) \tag{8}$$

c. The width of the bands measured on the image increases linearly with n.

[2] Ueber die Richtungsquantelung im Magnetfeld Ann. d. Physik, *76*, p. 185; 1926.

Denoting with S_n' and S_n'' the deflections for outer and inner edges of the nth band and substituting for v_n in (7) v_n' and v_n'' from the equation (4) we obtain for the edges of the n-th band the deflections

$$S_n'=\frac{1}{8}\frac{M}{m}\left(\frac{\delta H}{\delta S}\right)_0\frac{l^2}{D^2}\frac{(n-w/2\pi a)^2}{f^2}(K+1/2)$$
$$S_n''=\frac{1}{8}\frac{M}{m}\left(\frac{\delta H}{\delta S}\right)_0\frac{l^2}{D^2}\frac{(n+w/2\pi a)^2}{f^2}(K+1/2) \tag{9}$$

The difference of S_n'' and S_n' determines the width d_n of the n-th band. It follows therefore from (9)

$$d_n=S_n''-S_n'=\frac{1}{4}\frac{M}{m}\left(\frac{\delta H}{\delta S}\right)_0\frac{l^2}{D^2}\frac{1}{f^2}\frac{w}{\pi a}(K+1/2) \tag{10}$$

d. The relative intensity of each band will depend on the law of distribution of velocities within the ray. The latter can be determined by measuring the density of the bands on a number of images obtained with constant exposure and varying for each image the vibration frequency of the velocity selector.

It may be noted from the relation (8) that the determination of the magnetic moment of atoms with the proposed method of velocity selection requires neither the knowledge of the temperature of the source of the atomic rays nor that of the velocity. Further, the relation (10) leads to a new method of determination of magnetic moments by measuring the width of the bands d_n.

The sharpness of lines of an image produced in connection with the velocity selector is obtained chiefly by excluding all atoms whose velocity differs from those selected. There is, however, another factor, the reduction of the number of collisions between the atoms constituting the ray, which also increases the definiteness of the image. On account of the regulating action of the vibrating gate S_2, Fig. 1, the atoms move in the space S_2-S_3 in orderly groups. The distribution along the ray is such that most of the atoms of higher velocities proceed the slower atoms thus precluding the possibility of collisions.

How will the application of the velocity selector influence the time of exposure?

Let t_r be the time of exposure required for a non-deflected stream of atoms to produce an image of density d when the slits are at rest. With the slits vibrating a longer time of exposure t_v is necessary to produce the same density. During the time when 10,000 atoms emerge from resting slits a smaller number of only N atoms pass vibrating slits.

Furthermore the 10,000 undeflected atoms are distributed over a thickness of a line approximately equal to the width w of the slit, while $N/2$ deflected atoms are distributed over the thickness d_n of a selected line. In order to produce in both cases an equal density, equal numbers of atoms must be deposited per unit of area. Therefore,

$$10000\, t_r/w = N t_v/2d_n$$

and

$$t_v/t_r = 20000\, d_n/Nw \tag{11}$$

From equations (10) and (11) it follows that longer exposures will be needed for lines of larger n. However, for the determination of magnetic moments for velocities of atoms it will be sufficient to choose an exposure sufficiently long to obtain only one of the lines. It may be furthermore advantageous for the purpose of obtaining larger deflections, to adjust the frequency f of the vibrating slits for a velocity lower than for the most probable.

TABLE 1. *Data computed for an image deposited by rays of atomic hydrogen.*

n	v_n (m/sec)	v_n' (m/sec)	v_n'' (m/sec)	$\frac{100(v_n'-v_n'')}{v_n}$ (per cent)	N	S_n (mm)	d_n (mm)	$\frac{t_v}{t_r}$
0	∞		113040			0	0.000037	
1	3600	3720	3470	6.95	555	0.375	0.048	17
2	1800	1832	1767	3.61	152	1.500	0.095	125
3	1200	1215	1186	2.42	53	3.370	0.143	540
4	900	908	892	1.77	22	6.000	0.191	1740

Table 1 contains, besides the data mentioned above, also values for the deflection S_n, width of the lines d_n and the relative time of exposure t_v/t_r calculated for rays of atomic hydrogen emitted at a temperature of 500°K. The value for the Bohr magneton unit was assumed to be 5584 gauss cm, $(\delta H/\delta s)_0 = 140{,}600$ gauss per cm, and the length of the pole pieces of the deflecting magnet $l = 3.5$ cm. The value for K was assumed to be $\frac{1}{2}$ so that the correction for $(\delta H/\delta s)_l$, being different for each line and amounting to 1 to 5 per cent, was not considered. For the frequency of the velocity selector $f = 9000$ was chosen, for the width of the slits $w = 1/10$ mm, for the amplitude of the slit vibration $a = 1/2$ mm, and for the distance between the slits of the selector $D = 20$ cm.

UNIVERSITY OF ILLINOIS, URBANA, ILL.
JANUARY 15, 1927.

AN APPLICATION OF THE VACUUM TUBE OSCILLATOR

By C. B. Crofutt

A sustained source of sound is frequently desirable in certain experiments. The vacuum tube oscillator can be made to meet this requirement. When the metal rod in the Kundt's tube is replaced by the loud speaker of the vacuum tube oscillator, exceptionally good dust figures are obtained. The striations stand out with startling distinctness, the individual dust particles remaining suspended almost motionless in the gas. This may be used as a lecture demonstration or used in the laboratory, where a number of experiments will suggest themselves to the reader.

Fig. 1.

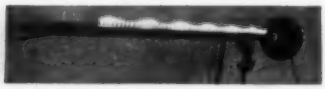

Fig. 2.

Figs. 1 and 2. *Photographs showing wave form assumed by flames.*

For lecture purposes it may be desirable to replace the glass tube with a brass tube having a row of small holes spaced about one-half inch apart along the tube to serve as gas jets. It is well to cover the end of the tube next to the loud speaker with rubber dam to prevent the escape of gas. If gas is fed into the tube and the row of gas jets lighted, the flames assume a wave form when the tube is in resonance, as shown in the photographs.

The vacuum tube oscillator used by the writer was built for general laboratory use and covered a frequency range from 200 cycles per second to 1,500,000 cycles per second. The Hartley circuit was used, with honeycomb coils for the inductances. For radio frequencies a variable air condenser was used. For audio frequencies a mica condenser built especially for the purpose was connected in parallel with the air condenser. The steps on the mica condenser were of such magnitude, as compared with the variable air condenser, that any capacity from the minimum of the air condenser to two microfarads could be obtained. By proper choice of capacity and honeycomb coils any desired frequency could be obtained. The tube used was the UX 112; however, it might be well to substitute a power tube. Connection was made to the loud speaker by means of a small audio transformer placed in the plate circuit.

This oscillator has been used successfully as a source of a. c. current for use on bridge circuits in electrical measurements, as a source of sound in the sound laboratory, and as a source of radio frequency oscillations in radio measurements.

The photographs were made by Hugh Jones ot the University of Arkansas.

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ORONO, MAINE.

Magnetic properties of iron crystals.—Discs about 4 mm in diameter and 0.3 mm in thickness were cut from two single crystals of fairly pure iron (analyses are given, showing the fractions of a per cent of Mn, P, and other elements) with the plane of the disc lying parallel to one of the 100 planes of the lattice. To measure the component of magnetization parallel to the field, the author begins by adjusting the applied field so that it lies in the plane of the disc and makes an angle θ with one of the tetragonal axes in this plane; he then displaces the direction of the field through 5° in the plane normal to that of the disc, and measures the torque necessary to restore the disc to its initial orientation; this measurement is repeated for each of a number of values of θ. To measure the component of magnetization perpendicular to the field, the author begins as before, but displaces the direction of the field in the plane of the disc instead of normally to it. The parallel component of magnetization fluctuates cyclically about a mean value as θ varies, passing through maxima when θ is an even-integer multiple, through minima when it is an off-integer multiple, of $\pi/4$. The perpendicular component fluctuates almost sinusoidally about mean value zero, through which it passes whenever θ is an integer multiple of $\pi/4$. The author then shows that if we postulate a certain curious type of molecular field within the crystal, the resultant of this field and the impressed field (with due allowance for the demagnetizing factor) will always lie parallel to the magnetization. The field in question must be supposed to be the resultant of two fields, each directed along one of the tetragonal axes lying in the plane of the disc, and each equal to a factor M multiplied by the fourth power of the cosine of the angle between that axis and the direction of the magnetization. The value of M in gauss is 620 for one of the discs, and 470 for the other (in the strongest fields which the author used). [W. L. Webster (Cavendish); Proc. Roy. Soc., *107A*, pp. 496–509; 1925.]

KARL K. DARROW

A GENERAL EXPRESSION FOR THE TEMPERATURE CORRECTION BASED ON REGNAULT'S METHOD

By C. H. Dwight

In the usual procedure the experiment (e.g., the determination of the latent heat of fusion of ice) is divided into three periods, during the first and third of which the substance is exposed solely to the influence of the surroundings, while during the second period it is subjected to the joint action of the surroundings and of the heating or cooling agent, the effect of the agent alone being finally sought. By means of a "Rate-of-Cooling Curve," plotted from data taken in the first and third periods, the temperatures observed in the second period may be corrected, the result being a "Corrected Cooling Curve" the ordinate of the horizontal portion of which is the "corrected final temperature."

It is obviously desirable to discover the relation between the corrected value of any observed temperature and the data taken, without the labor of plotting the curves. We shall determine the rate of cooling R'' for an average temperature t'' in the first experimental period and, later on, a rate R' for an average temperature t' in the third period. If t_r be the temperature of the surroundings, then by Newton's Law of Cooling the rate at any temperature t (not too far from t_r) will be given by $R=K(t-t_r)$, where the value of K is given by $(R''-R')/(t''-t')$. We shall call the first temperature read during the second period t_0 and assume it is correct. The other temperatures in this period are t_1, t_2, t_3, $\cdots t_n$, read every m minutes for n intervals. The correction to be applied to the temperature t_1 at the end of the first interval will be $C_1=mR_0'$, where R_0' is the average rate of cooling throughout the interval just ended; it will be the rate at the average temperature $t_0'=(t_0+t_1)/2$. The correction to be applied to t_2 will be $C_2=C_1+mR_1'$ $=m(R_0'+R_1')$ and hence for the nth temperature reading the correction will be

$$C_n=m(R_0'+R_1'+R_2'+\cdots+R_{n-1}')$$

and on substituting for the respective rates their equivalents by means of the relation $R=K(t-t_r)$ we obtain

$$C_n=mK\left[\frac{t_0+t_1}{2}+\frac{t_1+t_2}{2}+\frac{t_2+t_3}{2}+\cdots+\frac{t_{n-1}+t_n}{2}-nt_r\right]$$

$$=mK\left[\frac{t_0+t_n}{2}+\sum_1^{n-1}t_i-nt_r\right]$$

where all the quantities are known. It is better to calculate t_r from the relation $K=R''/(t''-t_r)$, since R'' and t'' are accurately known.

Example: In an actual experiment the rates of cooling at the temperatures 33.5° and 7.25° were respectively $+0.24$ and -0.19, giving $K=0.0164$ and $t_r=18.88°$; $t_0=32.30°$; minimum temperature to be corrected, $t_3=6.30°$, occurring $mn=1\times3=3$ minutes after t_0 was observed; the summation was $t_1+t_2=20.2°$.

Hence

$$C_3=1\times0.0164(19.3+20.2-3\times18.88)=-0.281$$

and hence the corected minimum temperature was

$$6.300+(-0.281)=6.019°,$$

a result that checked very well with the value obtained by actually plotting the curves. In this example the rates are in °C/min, and the temper- atures are in °C.

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X-rays and Electrons. By Arthur H. Compton, Professor of Physics, in the University of Chicago. XV+403 pages. D. Van Nostrand Company, New York. $6.00, net.

This volume is based on lectures on x-rays which the author has given for several years past. Greater emphasis is given to the recent theories of x-rays, and the relation of these theories to the structure of the atom, than to either crystal structure or the technique of spectroscopy, both of which are covered by other recent books.

The subject matter is divided into twelve chapters, viz.: I, The Discovery and Properties of X-rays; II, The Electromagnetic Theory of the Production of X-rays; III, The Scattering of X-rays; IV, X-ray Reflection and Crystal Structure; V, Intensity of the Reflection of X-rays from Crystals; VI, The Absorption of X-rays; VII, The Refraction of X-rays; VIII, The Photoelectric Effect with X-rays; IX, Quantum Theory of X-ray Scattering; X, Quantum Theory of X-ray Diffraction; XI, X-ray Spectra; XII, Quantum Theory of X-ray Production and Absorption.

There are also seven appendices containing certain numerical data and such mathematical material as "The Radiation from an Electron Moving with a Velocity Approaching That of Light."

The treatment throughout is characterized by a thorough-going discussion of theory, but yet with adequate emphasis on the experimental basis of theory. The numerous references to original sources will materially assist the critical reader in further study. Students of x-rays and of general physics will find this one of the most valuable books on the subject which has yet appeared. And it is "up to date," as is evidenced by the frequent references to work published in 1926. [F. K. R.]

A NEW TYPE OF PHOTOGRAPHIC EXPOSURE METER AND PHOTOMETER

By F. H. Norton

ABSTRACT

The development of a small and inexpensive photometer for determining light intensities and photographic exposures is described. A study is made of the performance of two types of instrument. In a large number of readings by a single observer with the smaller type it was found that the average error in measuring exposures on neutral colored objects was 22% which corresponds to about 7% difference in the density of the photographic image. The larger type had a precision of better than 5% for most of the range.

INTRODUCTION

It has long been recognized that the major cause of failures in making photographs has been a wrong estimate of the exposure. Also there are many photographs that may be classed as satisfactory but which fail of technical perfection because of a small misjudgment in the exposure.

When using an ordinary emulsion we have a considerable range of correct exposure. This is shown in Fig. 1 for a typical emulsion where

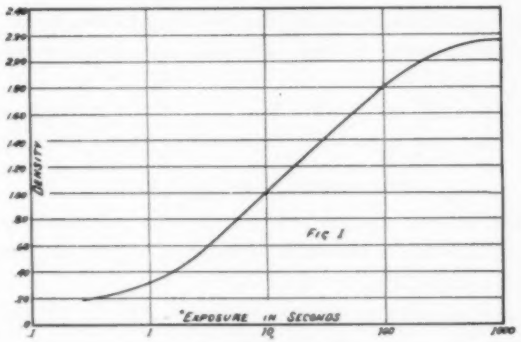

Fig. 1. *Typical density curve.*

the exposure is plotted against the density of the image in the usual way. The straight line portion represents the region of correct exposure and generally extends over a wide range. However, if the object to be photographed has considerable contrast it is necessary to restrict the mean exposure to the center of the straight line portion. In general an exposure correct to 100 per cent is sufficiently accurate, but in order to obtain the best results it is necessary to know the exposure required for both the shadows and high lights.

There have been many instruments used for measuring light intensity or exposure. One of the simplest measures the intensity of light by the time taken to darken special sensitive paper to a given shade. This type of instrument is small, compact and simple to operate, but it has the disadvantage of measuring the brightness of the light and not the brightness of the object. Also a long time is required to take a reading in a dim light.

Another type of instrument makes use of a small variable diaphragm placed close to the eye. This diaphragm is contracted until the eye cannot distinguish contrast in the field of view. This principle is not well adapted to accurate work because the sensitivity of the retina changes with the general light intensity and because the point where contrast disappears is not a very definite one with untrained observers.

Of course the most satisfactory type is a photometer where the brightness of the object to be photographed is compared with a standard source. Instruments of this type give excellent results, but have the disadvantage that they are rather bulky and heavy. The most portable type uses a calibrated electric lamp for a source but requires a rather large battery, an ammeter and an optical system.

The instruments described in the following paper are an attempt to produce a photometer having sufficient accuracy for measuring exposures; yet small enough to be carried in the pocket, and inexpensive enough for general use. This has been accomplished primarily by eliminating the ammeter, large battery and lens system.

PRELIMINARY DEVELOPMENT

Before the present design of instrument was evolved a number of relatively unsuccessful ones were tried out, a brief description of which will perhaps make the principles clearer.

Photometer type instruments, as shown in Figs. 2, 3 and 5, were tried but were inaccurate due to the necessity of matching low intensity lights of different colors and to the change in battery voltage. The latter error was corrected by having an adjustable rheostat scale, which operates as follows. Before taking a reading the rheostat is turned until, on looking directly at the light filament, it just ceases to be luminous. The scale is then shifted until the infinity mark on the scale is opposite the pointer or sliding contact. After this adjustment readings on the scale are correct until the voltage changes. This may be made clearer by referring to Fig. 4. The shifting of the scale is equivalent to changing the resistance in relation to the scale in such a way as to keep the

voltage across the lamp and portion of the resistance used for readings a constant regardless of the condition of the battery.

The error due to matching lights of different colors was reduced by the optical system shown in Fig. 6 which eliminated all screens between the object and the eye. The light from the lamp filament passes through a blue screen and is reflected by a 45° clear glass plate, and the object is viewed through this glass plate with no filter.

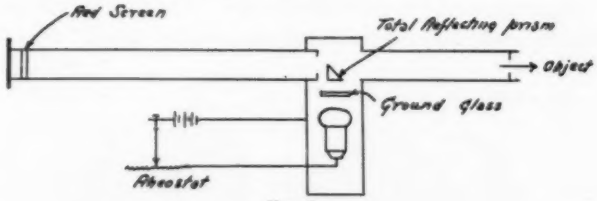

FIG. 2.

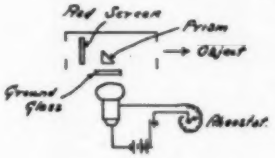

FIG. 3.

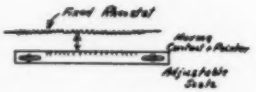

FIG. 4.

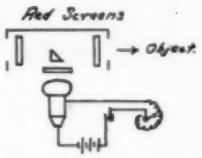

FIG. 5.

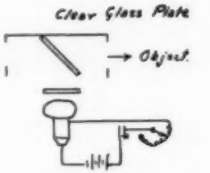

FIG. 6.

The method used here depends on the contrast sensitivity of the eye. The minimum discernable contrast is approximately proportional to the object brightness. Therefore the brighter the object viewed, the brighter the filament image must be for minimum visibility. The ratio of object brightness to minimum contrast is not exactly constant for the whole range of brightness, but this is unimportant if the instrument is calibrated at several points.

In use the blue image of the filament is reduced in brightness by the rheostat until it just disappears against the background of the desired object. It was found that with practice accurate settings could be made, but the system had the disadvantage that balancing was only in one

direction, that is, there was no indication of how much below the correct value the setting was, and this caused difficulty with some observers. Except for this fault the instrument was quite satisfactory; it could be rapidly balanced in any light without eye-strain, and was light and compact.

THE FINAL FORMS OF THE EXPOSURE METER

The next type of instrument constructed was similar to the preceding except that it had a more open scale and a more precise method of balancing. This method consists in substituting for the 45° clear reflector

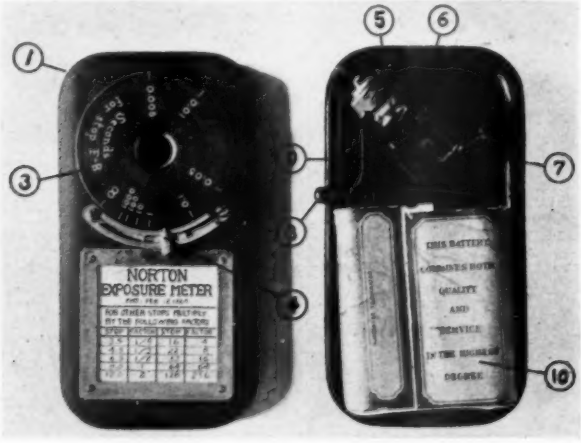

FIG. 7. *Small type of exposure meter.*

a piece of very lightly smoked glass. This reflector gives two images, one slightly brighter than the other, for one ray passes through the glass and is reflected from the back and the other (the brighter) is reflected from the front. In balancing, the image brightness is reduced until the weaker image is invisible and yet the stronger one can just be seen. This gives a far better point of balance than in the preceding instrument.

The details of this instrument are shown in Fig. 7. (1) is a brass case about the size of a pocket flashlight. The rheostat is mounted in a large graduated dial (3), and (4) is the adjustable reference mark. The flashlight bulb (5) is screwed into a socket and projects its light through the blue filter (6) onto the smoked glass reflector (7). The flashlight battery (10) sends current through the spring (9) to the rheostat, then

through the case to the lamp and back to the battery through the push button (8).

To use the instrument the reference pointer is first set by holding the instrument back up in a shadow and turning the rheostat until the directly viewed filament just ceases to be luminous, being careful to avoid bright reflections on the cover glass. The instrument is then turned over and the pointer is set at infinity. Now the instrument is held about 15″ from the eye and the desired object is viewed through the eye piece. Then the blue images are gradually dimmed until the weaker has disappeared leaving only the stronger. The correct exposure for F-8 and a standard make of film is then read directly off the scale. For other stops and emulsion speeds the reading can be multiplied by an appropriate factor. When a number of readings are to be taken in succession it is unnecessary to reset the pointer each time. It usually requires about 10 seconds to take a reading.

A tubular type of instrument was constructed later, having greater precision and convenience of balancing than the preceding type. A section of this instrument is shown in Fig. 7a. A lens is introduced

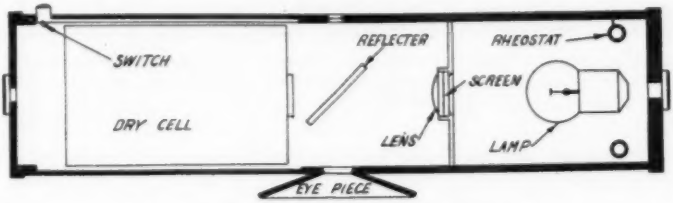

FIG. 7a. *Section of tubuler type of exposure meter.*

between the lamp and the screen so that the eye can be placed at the eye-piece and focus simultaneously on the filament and object. This has the advantage of limiting the light received by the eye to that from the desired object, which prevents errors when taking readings of poorly lighted objects with bright areas in the same direction.

Instead of a 3 cell battery, a single unit cell is used with a 2.5 volt lamp. Due to the use of the lens and a slightly lighter blue screen sufficient light is obtained from this combination. The voltage change of the battery is slight and its life comparatively long, due to its greater capacity and the smaller current drawn.

The rheostat is in a more convenient location than on the preceding model, which makes balancing easier. However, the instrument is somewhat more bulky.

A TEST OF THE FLAT TYPE OF INSTRUMENT

In order to set forth the characteristics and precision of this type of instrument, a more or less complete test of the model shown in Fig. 7 has been made and the results are given below.

In Fig. 8, there is shown the characteristics of the flash light battery (new) used when connected directly to the lamp bulb recommended for

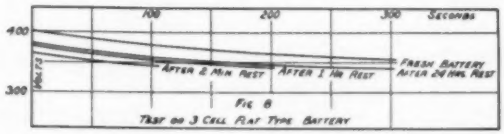

FIG. 8.

it, that is, 3.8 volts and about 0.3 amp. It should be noted that the variation of voltage with time is quite rapid immediately after closing the circuit.

In Table 1 there are given values of the voltage across the lamp for various positions of the rheostat.

TABLE 1.

Exposure indicated	Voltage across Lamp	Exposure Indicated	Voltage across Lamp
∞ secs.	.30 volts		
1000 "	.47 "	.1 secs.	.85 volts
100 "	.50 "	.05 "	1.03 "
10 "	.54 "	.01 "	1.91 "
1 "	.60 "	.005 "	2.25 "

It will be seen that the scale is crowded together at the lower end which is a disadvantage when measuring very long exposures, yet allows an accurate setting of the reference point at the ∞ mark.

A test was next made to determine the accuracy of setting the reference point under different light conditions. This was done by turning the rheostat until the filament just ceased to glow and then reading the voltage across the lamp. The results are given below:

TABLE 2.

Light	Voltage across lamp
Sunlight in shadow of body	.32
Bright room in shadow of body	.31
Very dim room	.30

The effect of the sensitivity of the eye on the reference setting is seen to be small but it is enough to affect the precision on long exposures. If the instrument is calibrated with the reference point set in a dim light then this error may be considered negligible.

Two similar flat type exposure meters were calibrated by first putting an arbitrary scale on the dials and then obtaining the readings on a large number of grey and colored surfaces in various lights—from bright sunlight to a very dim room. Photographs were taken with several exposures under each light condition on a standard film and the films were all developed alike in pyro. The density of each film was then measured by a Martin's Polarizing Photometer and a curve of density plotted against log. exposure for each case (that is, the same lighting but different exposures). The exposure time where each curve cut the density 1.00 was taken as correct. The exposures thus found were then plotted against the scale on the instrument.

The curves for the two instruments on colorless objects are shown in Fig. 9, the difference between them being due to the dissimilarity in the

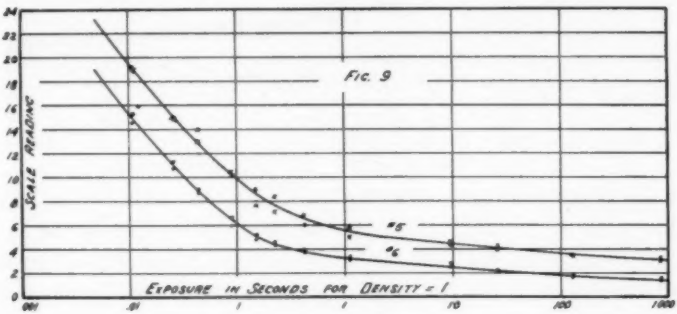

FIG. 9. *Calibration curves.*

lamp bulbs. The relation of the points to the mean curve gives a good insight into the precision of the instrument as all conditions of light are represented here. Summing up the deviations for each of the instruments gives the following results:

TABLE 3.

	Inst. No. 5	Inst. No. 6	Ave. of No. 5+6
Ave. deviation in exposure	27 per cent	16 per cent	21 per cent
Max. deviation in exposure	60	50	55
Ave. deviation in density	8	5	7
Max. deviation in density	18	15	16

The precision indicated is sufficient for exposure measurements even when it is conceded that the deviations would be larger if the test was made with a number of observers.

A similar test was made on colored surfaces and the curves obtained by one instrument are shown in Fig. 10. The pure reds and yellows are several hundred per cent too high as would be expected for a film of this type. On the other hand the deep blues are too low by a smaller amount. However, the lighter shades or impure colors show little difference from gray, so that it may be concluded that the majority of colors encountered will be measured correctly.

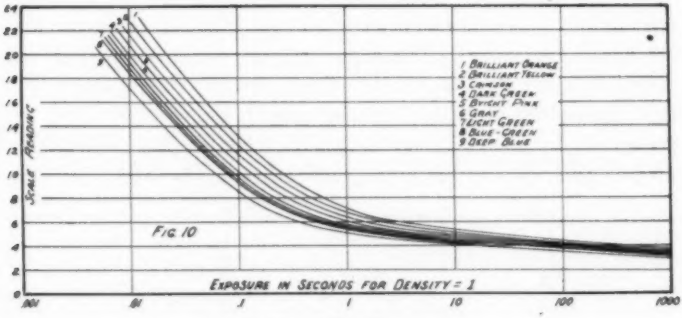

FIG. 10. *Calibration curves.*

A TEST OF THE TUBULAR TYPE OF INSTRUMENT

A test of the model shown in Fig. 7a was made which showed considerably greater precision than for the flat type. This is due to the ability to focus on the filament image and the object simultaneously, to the exclusion of all light not from the desired object, and to the more constant battery voltage.

The accuracy of the zero setting, that is the point where the directly viewed filament ceased to glow, was tested under various light conditions as shown in Table 4 below.

The precision of setting evidently increases as the light decreases and there is a difference of .45 divisions between a bright sunlight setting and a setting in darkness. This difference while not serious at high light intensities, causes considerable error in the low range. If the setting is always made in the darkest available place this error can be kept quite small, for generally where the instrument is used to read low intensities, the zero setting can be made under conditions approaching darkness.

TABLE 4.

Precision of zero setting.

Light conditions	Outdoors in sun but in shadow of body		In light room		In dim room		In darkness	
Scale readings	5.34	.13	5.40	.02	5.73	.00	5.87	.01
	5.18	.03	5.46	.04	5.78	.05	5.89	.03
	5.17	.04	5.45	.03	5.69	.04	5.81	.05
	5.16	.05	5.36	.06	5.73	.00	5.87	.01
	5.21	.06	5.42	.04	5.73	.02	5.86	.02
	Mean	A.D.	Mean	A.D.	Mean	A.D.	Mean	A.D.

The accuracy with which a setting could be duplicated when viewing an object through the eye piece in several different lights is shown in Table 5.

TABLE 5.

Precision of settings

Bright sun		Weak sun		Bright room		Very dim room	
5.42	.09	3.25	.09	1.58	.04	1.13	.07
5.55	.22	3.29	.05	1.57	.05	1.10	.04
5.15	.18	3.42	.08	1.72	.10	.95	.11
5.20	.13	3.40	.06	1.60	.02	.98	.08
5.35	.15	3.34	.07	1.62	.05	1.06	.07
Mean	A.D.	Mean	A.D.	Mean	A.D.	Mean	A.D.

A curve of scale readings on this instrument against light intensity in foot candles is shown in Fig. 11. From this we can derive the following

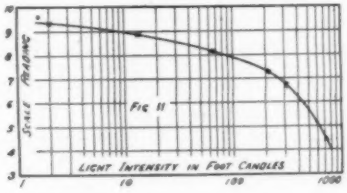

FIG. 11. *Calibration of larger instrument.*

probable error when using this instrument in different lights, assuming the calibration to be correct. It should be kept in mind that the percent-

age error would be somewhat larger if the readings were taken by a number of observers.

TABLE 6.

Precision of readings.

Light	Foot candles	Probable error
Bright sun	670	±2.8 per cent
Weak sun	530	±2.1
Bright room	32.5	±3.1
Very dim room	9.3	±6.6

I wish to express my indebtedness to Professor A. C. Hardy of the Physics Department for his help and suggestions in developing this instrument.

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THE EXTRUSION OF BISMUTH WIRE

By Donald C. Stockbarger

In the course of radiometric investigations during the past few years we have needed better materials for building thermopiles than copper and constantan. Considering thermoelectromotive force, electrical and thermal conductivities and ease of handling it appears to be believed generally that the bismuth-silver combination is the most satisfactory. Combinations yielding higher thermoelectromotive forces per degree temperature difference can be found but usually the gain is offset by high electrical resistance, and the junctions may be too fragile for practical use. For a good discussion of this matter, see a paper by Coblentz entitled "Various Modifications of Bismuth-Silver Thermopiles Having a Continuous Absorbing Surface."[1]

Bismuth cannot be drawn by ordinary means, and therefore a number of methods for producing wires have been offered from time to time. They have usually consisted in dropping the molten material from a height upon a smooth perfectly clean glass plate[2,3] whereby the metal was made to spatter out into a thin plate which could be rolled thinner and cut into narrow strips, or else it was made to spatter in the form of thin flat threads. Pfund[4] has described a similar method which enabled him to produce longer, finer filaments. Occasionally mention is made of fine round bismuth wires and a few years ago the writer saw some that were being used in the construction of scientific instruments. Evidently these had been produced by extrusion, but if anything was published concerning the method and apparatus it has escaped the writer's notice. Bridgman,[5] however, published a note concerning the extrusion of antimony and tellurium wires and thanks are due to him for suggestions which led to the development of the apparatus herein described. Employing a modification of Bridgman's apparatus the production of bismuth wires has been made easy and the results are so gratifying that it seems worth while to pass the idea along.

The extrusion press is shown diagrammatically in Fig. 1. All parts are made of high speed tool steel, accurately ground and lapped to size where required and carefully heat treated. The die block, Fig. 2, in use at present has an orifice 0.004 inch in diameter by 0.002 inch in length.

[1] Bull. Bur. Stds., *11*, p. 131; 1914.
[2] Bull. Bur. Stds., *7*, p. 243; 1910.
[3] Bull. Bur. Stds., *9*, p. 10; 1912.
[4] Phys. Rev., *34*, p. 228; 1912.
[5] Phys. Rev., *9*, p. 138; 1917.

The side of the cone of approach makes an angle of 45 degrees with the vertical axis. The dimensions are satisfactory for bismuth and bismuth-

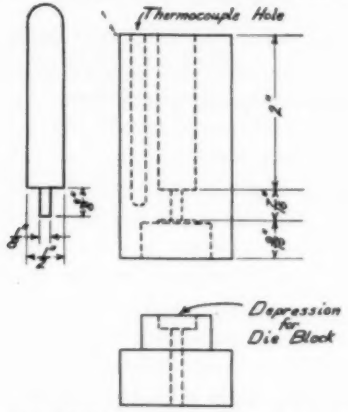

FIG. 1. *Sketch showing principal dimensions of extrusion piston and cylinder. The support for the die block shown beneath the cylinder can be screwed into the cylinder or held by machine screws.*

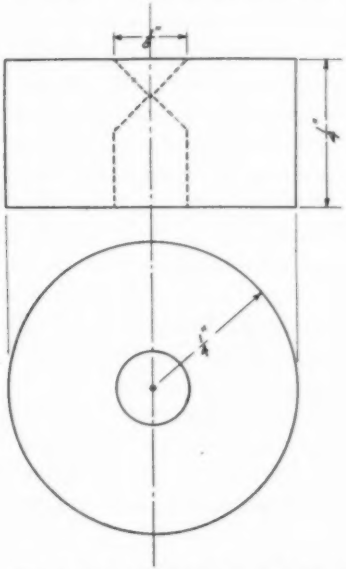

FIG. 2. *Die block. The size and shape of orifice as well as the angle of approach can be varied to suit the requirements.*

tin alloys but probably would have to be changed for some other materials.

In operation, either a small slug or else a quantity of powdered bismuth is placed in the 1/8 inch cylinder, the pistons and die block put in place, the whole electrically heated to a temperature somewhat below 100°C and a pressure applied on the end of the 1/2 inch piston by means of a hydraulic press. After a few minutes the wire begins to appear and continues to be extruded as a single piece until the capacity limit of the apparatus has been reached.

The bismuth wire is pliable so that it can be wound up on a spool as fast as it is formed. It can be wound around a 1/8 inch rod and straightened out again or bent several times in one place without breaking if care is exercised. It possesses good tensile strength but may break if bent under compression, i.e., if a short piece is bent by pushing on the ends it is almost certain to fail.

The wire can be extruded at room temperature but less pressure is required when the press is heated. The physical properties of the wire appear to depend upon the extrusion temperature, but no very reliable data have been obtained so far. It suffices to say for the present that wires made at temperatures from 25°C to 100°C have proven satisfactory mechanically. N. Johnston and S. F. Morgan, found in 1924 that the material would extrude somewhat explosively in the form of a powder when the temperature was too high and the pressure was applied suddenly.

They succeeded in extruding a piece of antimony wire several feet long and had no difficulty whatever in producing bismuth-tin alloy wires of various compositions up to 100 per cent tin. The antimony wire was exceedingly brittle, but the alloy wires were in general more pliable than those of pure bismuth. A number of them could be tied in tightly closed knots.

The preliminary experiments just mentioned and later ones in 1926 by M. W. Fort and P. V. Jewell indicated that there is an ageing effect in the case of at least some of the wires. It appears that the thermo-electromotive force per degree against copper, for example, may be different at the end of a day or more from what it was when the wire was first made.

Further studies are being made and will form the subject matter of later papers.

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NOTE ON THE PRODUCTION OF FLUORITE IN THE ELECTRIC FURNACE

By Donald C. Stockbarger

Crystalline calcium fluoride, commonly known as fluorite, of high transparency to ultraviolet of short wave lengths is rare. It seemed advisable to attempt, as doubtless many others have done, to produce it in the laboratory in pieces suitable for lenses, small windows, etc. With this in view preliminary experiments were performed in 1922 by G. E. Dean and R. D. Stuart. These indicated that the material could be melted successfully but the solidified mass was opaque.

During the past year C. S. Draper attacked the problem anew. Chemically pure calcium fluoride was fused in an atmosphere of nitrogen in a vertical electric graphite tube furnace. In order to cool the melt slowly the crucible was lowered through the tube from the hottest to the coolest region by a clock mechanism. Unfortunately it was impossible to maintain constant temperature in each region of the furnace, but even under these unfavorable conditions it was found easily possible to produce pieces composed of crystals which could be distinguished with a pocket magnifying glass. Improvements in manipulation led to conditions slightly less unfavorable under which the solidified mass as a whole resembled a single large crystal with numerous fractures along its cleavage planes. The small crystals, which were about one sixteenth inch across adhered strongly. The whole in the hands of a geologist was readily cut in two along a cleavage plane.

The work thus far has been so encouraging that we are fairly confident that better temperature control will enable us to grow large perfect crystals. Further experimentation has been begun in a molybdenum wound tube furnace in which the temperature variation is small.

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